Pre-Calculus Name: Date: Period:

5C: Systems of Inequalities with 2 Vars. - *Solutions*

Graphing Systems of Inequalities

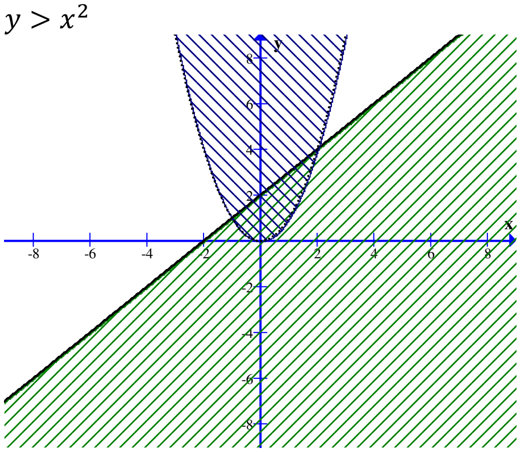
Steps for graphing inequalities:

1. Graph the equation represented by the inequality
2. Decide if it's dotted ($<$ *or* $>$) or solid ($\leq$ *or* $\geq$)
3. Test point(s) in the inequality to find the shaded region.

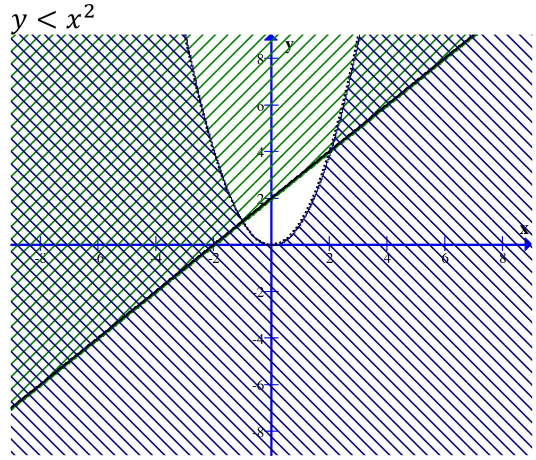
Systems of Inequalities: graph all inequalities and shade the overlapping regions, this represents the solutions to the system.

Example Solve the system of inequalities by graphing.

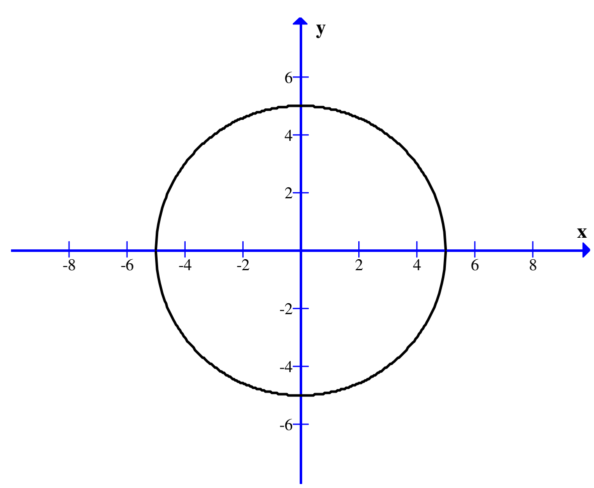
$y \leq x + 2$
$y > x^2$



$y \geq x + 2$
$y < x^2$

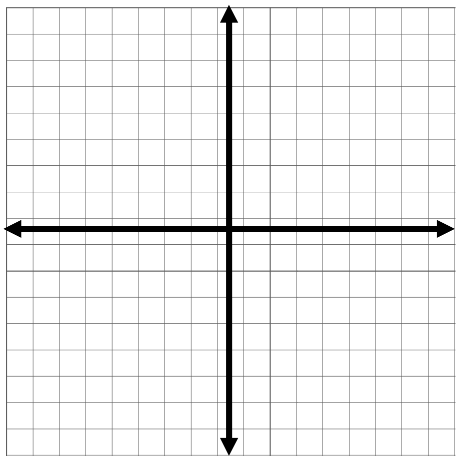


Explore Find all the integers coordinates (x, y) that satisfy the equation $x^2 + y^2 = 5^2$.
(Hint: think about the Pythagorean Theorem.)
Graph these points.
What shape would you get if you find all the non-integer solutions as well?

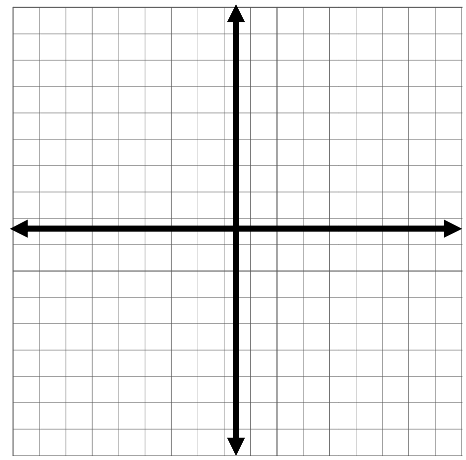


The **equation of a circle** centered at the origin with radius r is : $x^2 + y^2 = r^2$
Example Solve the system of inequalities by graphing.

$x^2 + y^2 \le 36$
$y > x - 3$
$y < x + 3$



$x^2 + y^2 > 4$
$y \ge x$



Linear Programming is the process of graphing a system of inequalities and finding the solution that maximizes an ***objective function***. The inequalities are called the ***constraints*** and the overlapping region is called the ***feasible region***.

The amazing result of linear programming is that the maximum or minimum value of the objective function occurs at one of the ***vertex points*** of the feasible region.

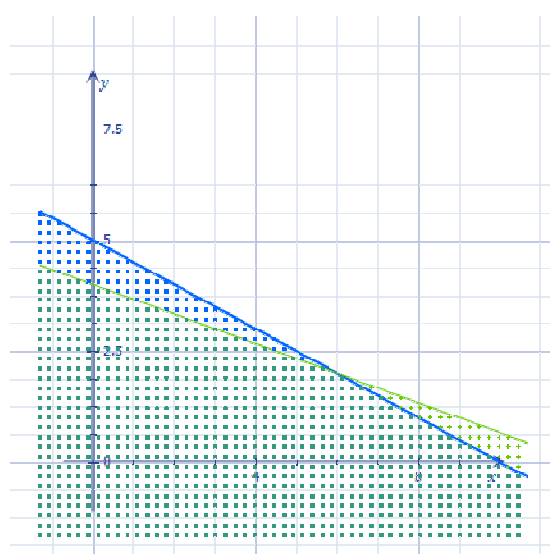
Example Find the maximum and minimum for the objective function with the given constraints.

Constraints:

$2x + 4y \le 20$
$x + 3y \le 12$
$x \ge 0, \;\; y \ge 0$

Objective Function: $f = 4x - 3y$

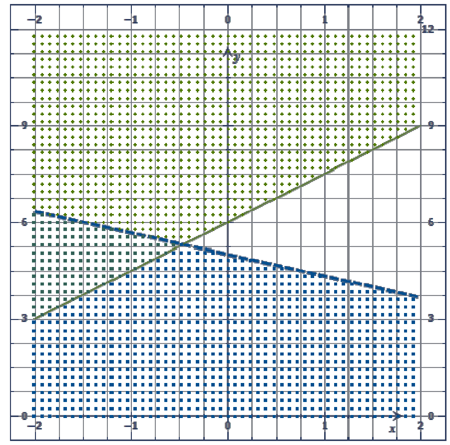
$f_{min} = -12 \; at \; (0,4)$
$f_{max} = 40 \; at \; (10,0)$



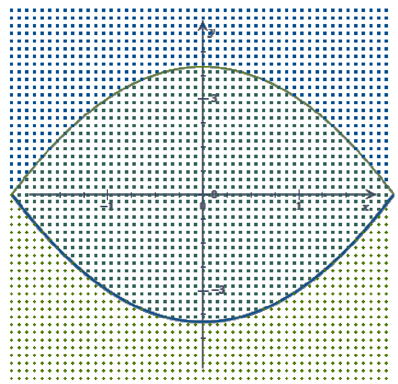
Exercises

Graph the system of inequalities and shade the solution.

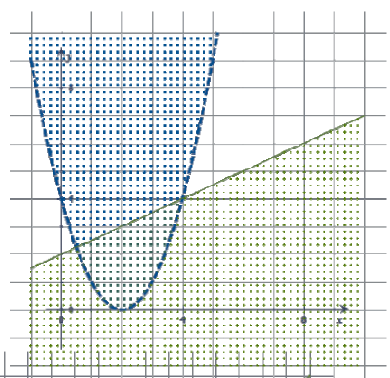
1. $2x + 3y < 15$
   $-6x + 4y \geq 24$



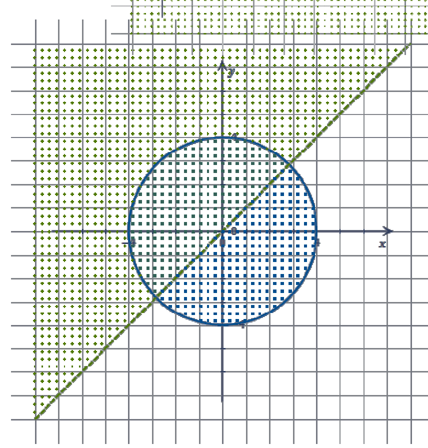
2. $y \geq x^2 - 4$
   $y \leq -x^2 + 4$



3. $y > (x - 2)^2$
   $y \leq \frac{1}{2}x + 2$



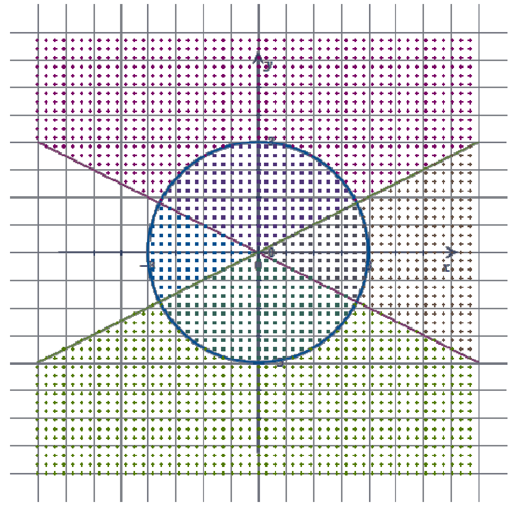
4. $x^2 + y^2 \leq 16$
   $y > x$



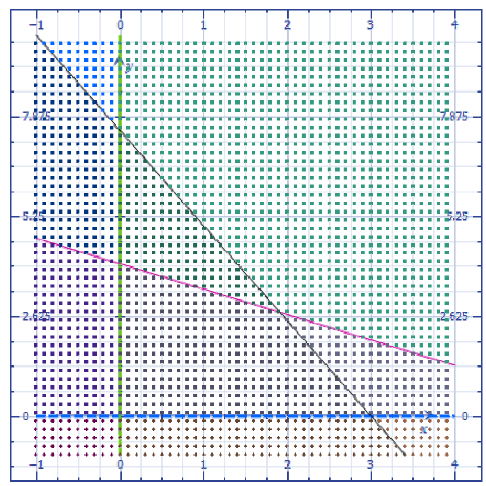
5. $x^2 + y^2 \leq 9$

   $y \leq \frac{1}{2}x$

   $y \geq -\frac{1}{2}x$



Graph the inequalities to find the feasible region. Then find the maximum and the minimum of the objective function, if they exist, subject to the constraints.



6. Constraints:

   $y > 0$

   $x > 0$

   $2x + 3y \leq 12$

   $5x + 2y \leq 15$

   Objective function: $f = 10x + 5y$

   Solution: Corners $= (\mathbf{0}, \mathbf{0}), (\mathbf{0}, \mathbf{4}), \left(\frac{\mathbf{2}}{\mathbf{11}}, \frac{\mathbf{30}}{\mathbf{11}}\right), (\mathbf{3}, \mathbf{0})$

   $f_{min} = \mathbf{0}\ at\ (\mathbf{0}, \mathbf{0}),\qquad f_{max} = \frac{\mathbf{170}}{\mathbf{11}}\ at\ \left(\frac{\mathbf{2}}{\mathbf{11}}, \frac{\mathbf{30}}{\mathbf{11}}\right)$

7. Constraints:

   $5x + y \geq 60$

   $x + 6y \geq 60$

   $4x + 6y \geq 204$

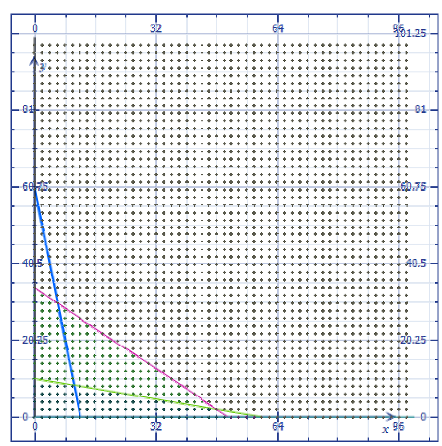
   $x \geq 0$

   $y \geq 0$

   Objective function: $f = 7x + 4y$

   Solution: Corners $= (\mathbf{0}, \mathbf{60}), (\mathbf{6}, \mathbf{30}), (\mathbf{48}, \mathbf{2}), (\mathbf{60}, \mathbf{0})$

   $f_{min} = \mathbf{162}\ at\ (\mathbf{6}, \mathbf{30}),\qquad f_{max} = none$



8. A manufacturer wants to maximize the profit for two products. He has provided you with the following information about his products:
   - Product A yields a profit of $2.25 per unit, and product B yields a profit of $2.00 per unit.
   - Demand information requires that the total number of units produced be no more than 3000 units.
   - Also, the number of units of product B produced must be greater than or equal to half the number of units of product A produced.

   Define your variables, write an objective function to maximize profits, and write a system of constraints. Then find out how many of each unit should be produced to maximize the profit.

Solution:

$x = units\ produced\ of\ Product\ A$

$y = units\ produced\ of\ product\ B$

$x + y \leq 3000$

$y \geq \frac{1}{2}x$

$x \geq 0, y \geq 0$

$Profit = P = 2.25x + 2.00y$

Vertex Points=$(0, 3000), (2000, 1000), (0, 0)$

$P_{max} = \$6500\ at\ (2000, 1000)$