Pre-Calculus

Unit 5 Toolkit – Systems of Equations & Inequalities

This toolkit is a summary of some of the key topics you will need to master in this unit.

5A: Systems of Equations With 2 Variables

Learning Target: I can solve and apply systems of linear and nonlinear equations in two variables.

A **solution of a system** is the ordered pair (for a 2 variable system) or ordered n-tuple (for a system with n variables) that is a solutions for all equations in the system.

Methods for solving 2 variable systems:

I. Graphing

Graph both equations and find the coordinates of their intersections

II. Substitution

1. Solve one equation for one variable,
2. Substitute into the second equation,
3. Solve for first variable, and
4. Re-substitute this first solution to find the second variable.

III. Elimination

1. Algebraically arrange both equations to have the same form,
2. Add or subtract (or sometimes multiply or divide!) the equations to eliminate a variable.
3. Solve for first variable.
4. Re-substitute this first solution to find the second variable.

IV. Matrices: Gauss-Jordan Elimination

1. Write system as an augmented matrix $\left[\begin{array}{cc|c} * & * & * \\ * & * & * \end{array}\right]$
2. Use elementary operations to get into row-echelon form $\left[\begin{array}{cc|c} * & 0 & * \\ 0 & * & * \end{array}\right]$
3. Divide each row to get reduced row-echelon form $\left[\begin{array}{cc|c} 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{array}\right]$
4. Write solution $x = a_1, y = a_2$.

Elementary Row Operations:
When changing a matrix into *row echelon form*, we are may use any of the following operations:

1. Interchange two rows.
2. Multiply a row by a nonzero number (called a "scalar").
3. Add a constant multiple of one row to another.

Matrices on your TI-8x calculator. You can use your TI-8x graphing calculator to find the reduced row echelon form of an augmented matrix with the following steps:

1. Go to the Matrix menu by hitting [2nd], [MATRX].
2. Go to EDIT and choose [A], change the dimensions to 2 x 3, and type in the values
3. Go to the Matrix menu, choose MATH, and select "rref("
4. Now go back to the Matrix menu and select the matrix name [A], then press enter to get the reduced row echelon form.

5B: Systems of Linear Equations in Three Variables

Learning Target: I can solve and apply systems of linear equations in three variables using substitution, elimination, and matrices

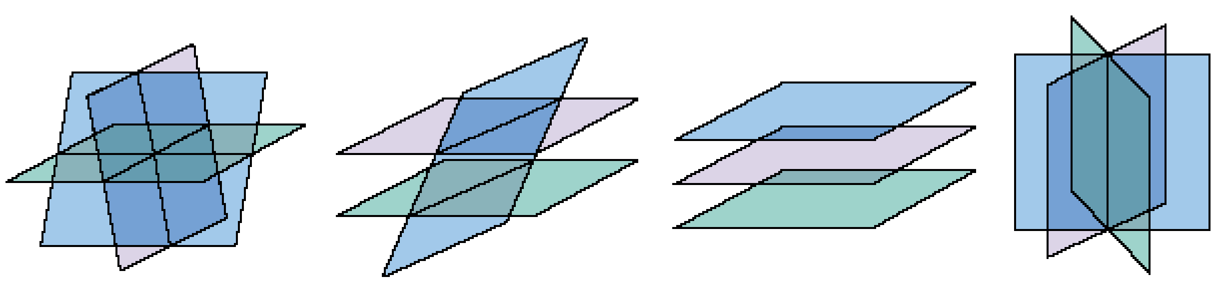
<u>Methods for solving 3 variable systems:</u>

Here are some methods for solving a system of 3 variables with 3 equations. The solution to such a system is an ordered triple (x, y, z) if there is a unique solution.

I. **Substitution:**
   1. Solve one equation for one variable in terms of all the others.
   2. Substitute this equation into the previous equations. This will leave you with 2 equations with 2 unknown variables.
   3. Now solve this smaller system for the first 2 variables.
   4. Back-Substitute to find the value of the third variable.

II. **Gaussian Elimination**
   1. Using pairs of equations, eliminate variables until the equation is in row echelon form such as $\begin{cases} x + b_1 y + c_1 z = d_1 \\ \quad\quad y + c_2 z = d_2 \\ \quad\quad\quad\quad z = d_3 \end{cases}$
   2. Back-Substitute to find the value of all variables.

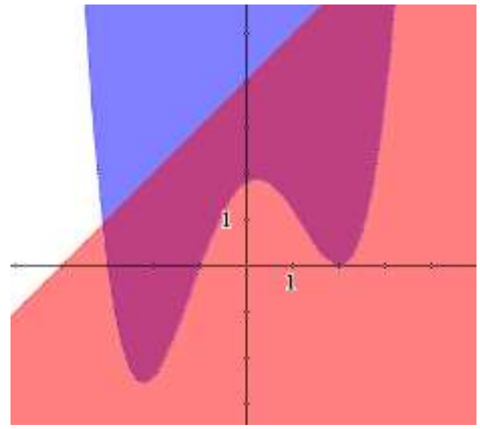
III. **Matrices**
   1. Write system as an augmented matrix $\left[\begin{array}{ccc|c} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{array}\right]$
   2. Use elementary operations to get into row-echelon form $\left[\begin{array}{ccc|c} * & 0 & 0 & * \\ 0 & * & 0 & * \\ 0 & 0 & * & * \end{array}\right]$
   3. Divide each row to get reduced row-echelon form $\left[\begin{array}{ccc|c} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 1 & a_3 \end{array}\right]$
   4. Write solution $x = a_1, y = a_2, z = a_3$



5C: Systems of Non-Linear Inequalities in Two Variables

Learning Target: I can solve and apply systems of nonlinear inequalities in two variables.

Graphing Inequality Functions:



1. Graph the function as an equation and decide if it is solid or dotted:
    - If operator is ≤ or ≥, the curve is solid.
    - If operator is < or>, the curve is dotted or dashed.
2. Shade one side of the curve:
    - Test a point on each side of the curve in the original inequality and shade toward the point the satisfies the inequality.
3. For systems, find the overlapping shaded region.

Linear Programming

1. Graph all inequalities and find the overlapping shaded region. This is the feasible region.
2. Find the coordinates of the vertices of the feasible region.
3. Evaluate the Objective function for each of the coordinates. This will find the minimum and maximum values of the objective function.