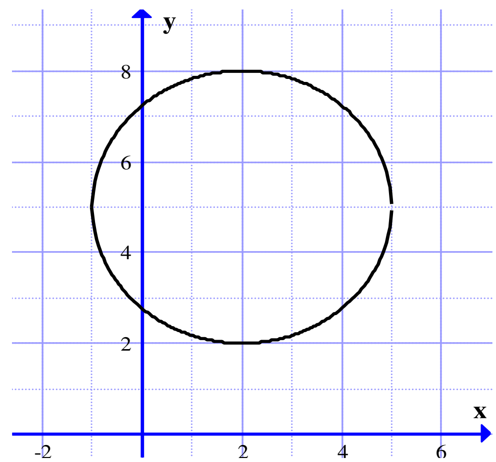
6A: Stretching Circles

Circles

A circle is the set of points that are equidistant from a center point. This distance is called the radius. For example, the circle to the right is the set of all points that are 3 units away from the point (2,5).



We can measure distance on the Cartesian plane by the distance formula (which comes from the Pythagorean Theorem).

So we know the points on this circle are all the points (x, y) that are 3 units from the center $(2,5)$. So, we can describe the circle with the

$$\sqrt{(x-2)^2+(y-5)^2} = 3$$

Squaring both sides gives us

$$(x-2)^2+(y-5)^2 = 9$$

Standard Form Equation of a Circle

In general, the **standard form equation of a circle** with radius r and center (h, k) is

$$\boldsymbol{(x-h)^2+(y-k)^2 = r^2}.$$

Try These (Solutions on next page)

Find the coordinates of the vertex and the length of the radius. If the equation is not in standard form, first change it into this form.

a) $(x+12)^2+(y-13)^2 = 144$

b) $x^2+(y-4.5)^2-5 = 5$

c) $x^2+6x+y^2-10y = 2$
(Hint: Complete the square for $x's$ and again for the $y's$ to rewrite the equation in standard from.)

Try These - Solutions:

a) Since the equation is in standard form the center is $(-12,13)$ and the radius in $\sqrt{144} = 12$.

b

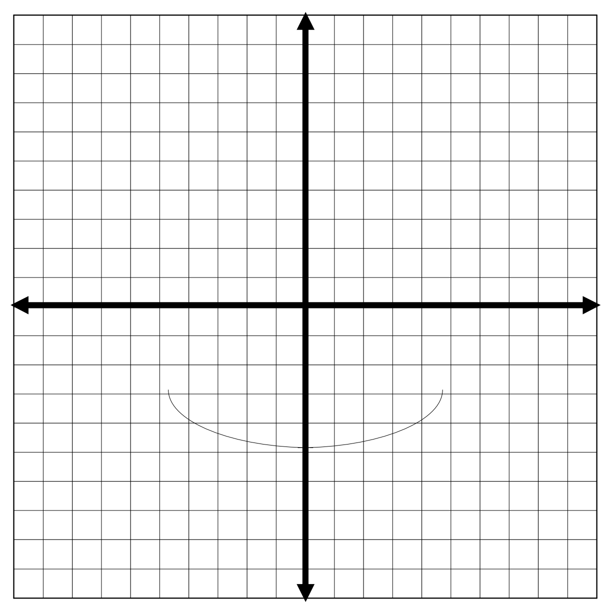
b) Add 5 to each side to rewrite the equation as $x^2 + (y - 4.5)^2 = 10$.
The center is $(0,4.5)$ and the radius is $\sqrt{10}$.

c) As stated in the hint, we need to complete the square twice to rewrite in standard form.
Remember, to complete the square for an expression $x^2 + bx$ we need to add $\left(\frac{b}{2}\right)^2$.

$$x^2 + 6x + y^2 - 10y = 2$$
$$x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 10y + \left(-\frac{10}{2}\right)^2 = 2 + \left(\frac{6}{2}\right)^2 + \left(-\frac{10}{2}\right)^2$$
$$(x^2 + 6x + 9) + (y^2 - 10y + 25) = 36$$
$$(x + 3)^2 + (y - 5)^2 = 36$$
$$Center: (-3,5), \qquad Radius = 6$$

Practice Problems 1

Write each equation in standard form for a circle and sketch their graphs on the same axe to the right.



1. $(x - 3)^2 + (y - 2)^2 - 4 = 0$

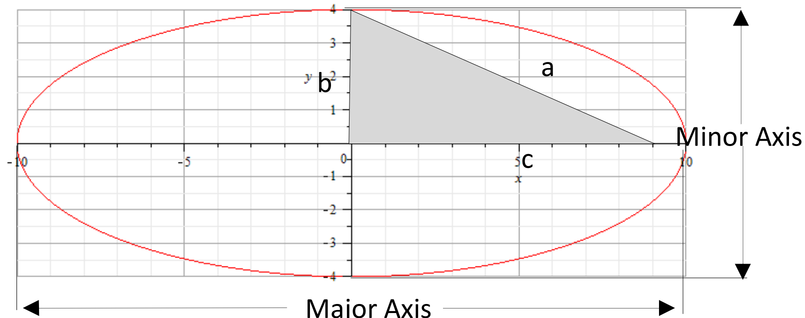
2. $x^2 + 4 = 5 - (y + 2)^2$

3. $x^2 + 6x + y^2 - 4y = -9$

4. $x^2 + y^2 + 4y - 60 = 0$

If you graphed them correctly, they should make you happy! =)

From Circles to Ellipses



While many things in our world are made circles, the governing shape of all objects traveling through space is the ellipse. An ellipse is the set of all points in a plane whose distances from *two fixed points (called the* ***"foci"*** *– plural for focus)* have a constant sum.

The **major axis** is the longest chord of the ellipse, and the **minor axis** is chord on the perpendicular bisector of the major axis.

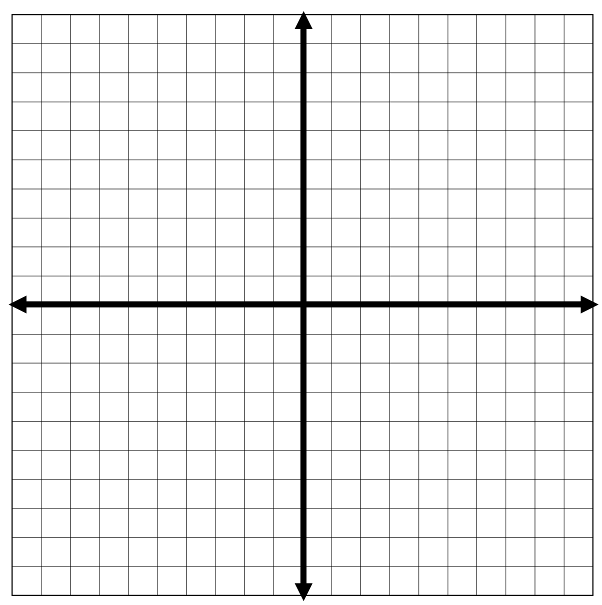
<u>**Explore**</u>

Let's explore to see how the equation for an ellipse relates to the equation of a circle.

1. Consider the equation for a circle $x^2 + y^2 = 16$. If we divide both sides by 16, we get the form

$$\frac{x^2}{16} + \frac{y^2}{16} = 1.$$

   a. What are the coordinates of the the $x - intercepts$?
   b. What are the coordinates of the the $y - intercepts$?
   c. Sketch the graph of this equation on the grid to the right.



2. Let's try modifying one of the denominators

$$\frac{x^2}{64} + \frac{y^2}{16} = 1$$

   a. What are the coordinates of the $x - intercepts$?
   (Hint: substitute 0 for y and solve for x)

   b. What are the coordinates of the $y - intercepts$?
   (Hint: substitute 0 for x and solve for y)

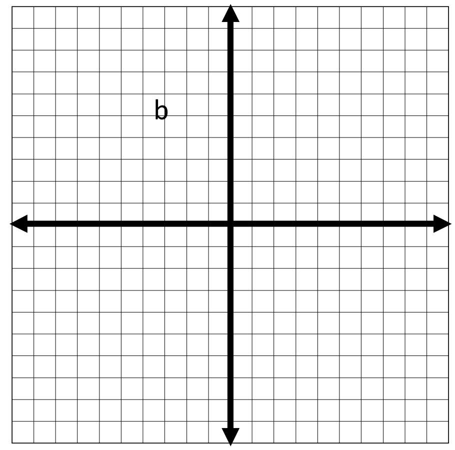
   c. Sketch the graph of this equation on the grid to the right.

   d. Find the length of the major and minor axes

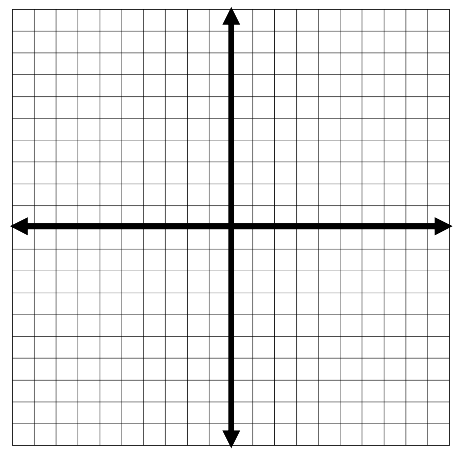
Practice Problems

For each of the ellipses below, graph them and find the length of the major and minor axis.

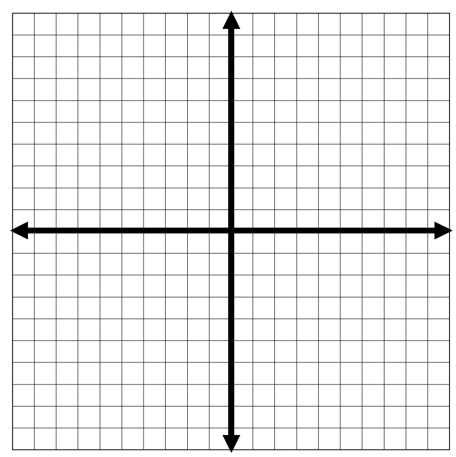
1. $\frac{x^2}{36} + \frac{y^2}{25} = 1$



2. $\frac{x^2}{4} + \frac{y^2}{49} = 1$



3. $\frac{x^2}{25} + \frac{y^2}{10} = 1$



4. $\frac{x^2}{40} + \frac{y^2}{81} = 1$