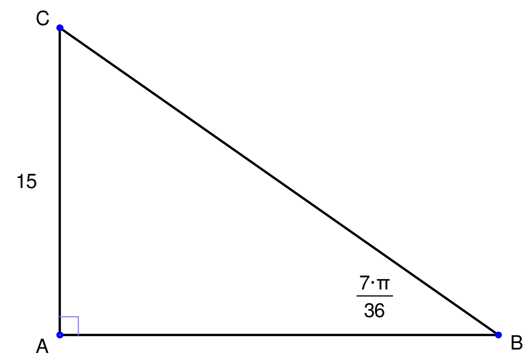
7A/B: Applications of Trig. Functions

Finding Missing Side Lengths in Right Triangles

We have learned that the six trigonometric ratios tell us the ratio of two sides in a right triangle. Now, let's use them to solve problems about missing sides.

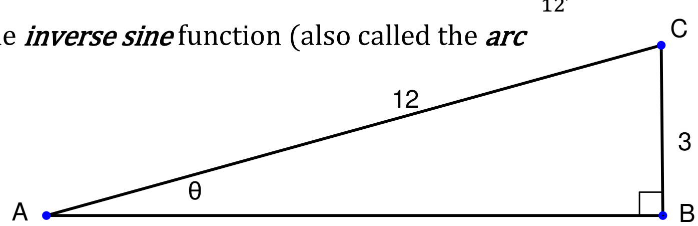
Example Find the length of the missing sides in the triangle to the right.



Finding Missing Angle Lengths in Right Triangles

Suppose we know the sides of a right triangle, but we need to know what the angle is. This is where we need to go in "reverse" and find the angle from the ratio.

Suppose he have the triangle to the right and we want to find θ. We know that $\sin\theta = \frac{3}{12}$, but what angle is this? To solve this, we need the ***inverse sine*** function (also called the ***arc sine***) to "undo" the sine and solve for θ.



$$Inverse\ Sine = arcsine = \sin^{-1} r = \text{"}What\ angle\ has\ a\ sine\ of\ \boldsymbol{r}?\text{"}$$

Likewise, we have six inverse trig. functions to find the value of an angle:

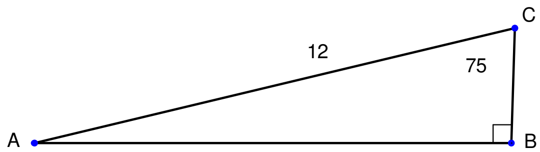
$$\sin^{-1} r, \quad \cos^{-1} r, \quad \tan^{-1} r, \quad \csc^{-1} r, \quad \sec^{-1} r, \quad \cot^{-1} r$$

Example Solve the triangle above. (This means to find all the missing sides and angles).

Practice Problems

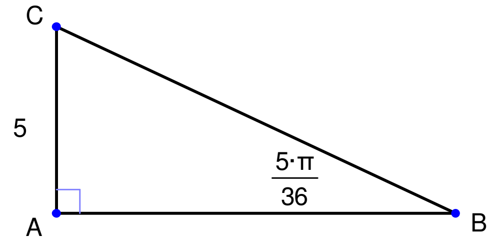
Solve the triangle by finding all angle and side measurements.

1. Angle is in degrees.

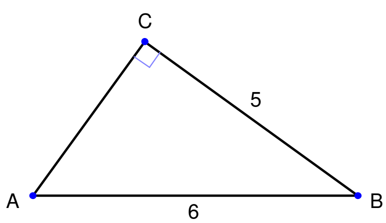


$$m\angle A = 15°$$
$$AB = 12\sin 75 = 11.59$$
$$C = 12\cos 75 = 3.11$$

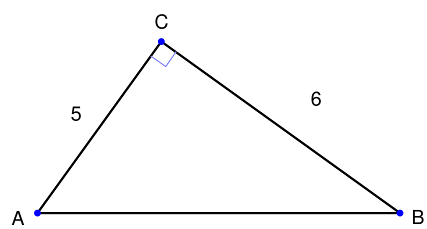
2. Angle is in radians



$$m\angle C = \frac{13\pi}{36}$$
$$BC = \frac{5}{sin\frac{5\pi}{36}} = 11.831$$
$$B = \frac{5}{tan\frac{5\pi}{36}} = 10.723$$



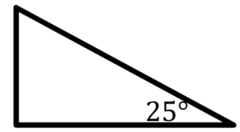
$$m\angle A = sin^{-1}\frac{5}{6} = 56.443$$
$$m\angle B = cos^{-1}\frac{5}{6} = 33.557$$
$$AC = \sqrt{6^2 - 5^2} = \sqrt{11}$$



$$AB = \sqrt{5^2 + 6^2} = \sqrt{61}$$
$$m\angle A = tan^{-1}\frac{6}{5} = 50.194°$$
$$m\angle B = tan^{-1}\frac{5}{6} = 39.806°$$

3. Two airplanes leave the Medford airport at the same time, one heading due west, and the other left at 25° north of west. After 2 hours, the first plane (heading west) had traveled 1000 miles and the second plane was exactly due north of the first plane.

    a. Draw a diagram of the airplanes' trip.



1000mi.

    b. How far did the 2^{nd} airplane travel to the nearest 100^{th} of a mile?

$$\boldsymbol{x = \frac{1000}{\cos 25} = 1103.378}$$

    c. Find the average speed of both airplanes.

$$\boldsymbol{Plane\ 1 = 500mph, \qquad Plane\ 2 \approx 551.7\ mph}$$

4. Two other planes leave the Medford airport. One is traveling due south at an average of 525 *mph*, and the other is traveling east of south at an average of 570 *mph*. After 2 hours, the second plane is due east of the second plane. What was the angle *between* the two planes when they left the airport (assuming that they have been flying a straight path).

$$\boldsymbol{\cos^{-1}\frac{1050}{1140} = 22.92°}$$

5. Caleb is flying a kite and he has let out 300 feet of string. His sister Emily is 50 feet away and standing directly underneath the kite.

    a. How far is Emily from the kite?

$$\boldsymbol{\sqrt{300^2 - 50^2} = 295.8ft.}$$

What is the angle that the kite string is making with the horizon?

$$\boldsymbol{\theta = \cos^{-1}\frac{50}{300} = 80.406}$$

    b. What assumption(s) do we have to make about this situation for these answers to be accurate?

We are assuming that the string is straight.
We are assuming