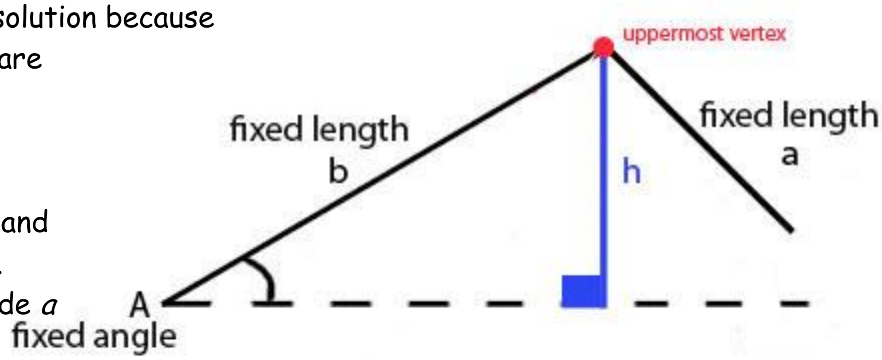
7D.1-SSA… The Ambiguous Case

When we have a Side-Side-Angle situation, the law of Sines is not guaranteed to give us the *only* solution because when we take the inverse sine, there are more than one possible angle.



For an acute angle *A*

Suppose we are given angle *A*, side *b*, and side *a* like in the drawing to the right. This will only make a triangle if the side *a* is longer than the height *h*. As we have seen before, the height can be found using $h = b\sin A$. So, we get three cases:

Relationship of a and $h = b\sin A$	Number of Triangles
$b\sin A > a$	0 possible triangles
$b\sin A < a \quad and \quad b < a$	1 possible triangle
$b\sin A < a \quad and \quad b > a$	2 possible triangles
$b\sin A = a$	1 possible triangle… it's a right triangle!

For an obtuse angle *A*

When we have an obtuse angle A, we simply need the opposite side a to be greater than b.

$a < b$	0 possible triangles
$a > b$	1 possible triangle

Extra Practice

1. Decide if there are one, two, or no possible triangles with the given measurements. Explain and show any work.

a. $a = 9, b = 7, A = 108°$

$\angle A$ is Obtuse, and $a > b$, so there is 1 possible triangle

b. $a = 14, b = 15, A = 117°$

$\angle A$ is obtuse, and $a < b$, so there are no possible triangles

c. $a = 5, b = 12, A = 27°$

$\angle A$ is acute

$$h = b\sin A = 12\sin 27 \approx 5.45$$

There are 0 triangles because $a < h$.

d. $a = 35, b = 24, A = 82°$

$\angle A$ is acute

$$h = b \sin A = 24 \sin 82 \approx 23.0$$

There is one triangle because $\boldsymbol{a > h}$ and $\boldsymbol{a > b}$

e. $a = 19, b = 38, A = 30°$

$\angle A$ is acute

$$h = b \sin A = 38 \sin 30 = 38 \cdot \frac{1}{2} = 19$$

There is one triangle (a right triangle) because $\boldsymbol{a = h}$.

f. $a = 6, b = 6, A = 63°$

$\angle A$ is acute

$$h = b \sin A = 6 \sin 63 \approx 5.01$$

There is one triangle because $\boldsymbol{b > h}$ and $\boldsymbol{a = b}$. It's Isosceles!

g. $a = 10, b = \sqrt{200}, A = 45°$

$\angle A$ is acute

$$h = b \sin A = \sqrt{200} \sin 45 = \sqrt{200} \cdot \frac{\sqrt{2}}{2} = 10$$

There is one right triangle (a 45-45-90) because $\boldsymbol{a = h}$.

2. Find two triangles with the given angle measure and side lengths. Round side lengths to the nearest tenth and angle measures to the nearest degree.

$$A = 39°, a = 12, b = 17$$

$$\frac{\sin 39}{12} = \frac{\sin B}{17}$$

$$B = \sin^{-1} \frac{17 \sin 39}{12} \approx 63.07°$$

$$or \ldots B \approx 180 - 63.07 = 116.93°$$

If $B = 63.07°$ $C = 180 - 39 - 60.61 = 77.92$ $\frac{\sin 77.92}{c} = \frac{\sin 39}{12}$ $c = \frac{12 \sin 77.92}{\sin 39} \approx 18.64$ **Solution 1:** $\boldsymbol{B = 63°, C = 78°, c = 18.6°}$	If $B = 116.93°$ $C = 180 - 39 - 116.93 = 24.07$ $\frac{\sin 24.07}{c} = \frac{\sin 39}{12}$ $c = \frac{12 \sin 24.07}{\sin 39} \approx 7.77$ **Solution 2:** $\boldsymbol{B = 116.93°, C = 24.06°, c = 7.77}$