Pre-Calculus Name: SOLUTIONS Date: Period:

Unit 7 Practice Test
Learning Targets: 7A, 7B, 7C, and 7D

Complete the problems below, show your work, and write your answer in the blank provided.

Target 7A: I can accurately use radian measures to define angles and find arc lengths.

1. Convert the given angle into the given measure.
   a. $45°36'10"$ to degrees

$$45 + \frac{36}{60} + \frac{10}{3600} = 45.6°$$

   b. $\frac{7\pi}{15}$ $radians$ to degrees

$$\frac{7\pi}{15} \cdot \frac{180}{\pi} = 84°$$

   c. $235°$ to radians

$$235 \cdot \frac{\pi}{180} = \frac{47\,\pi}{36} rad.$$

2. Find the arc length of a $35°$ arc that has a radius of $3cm$

$$s = \frac{\pi r\theta}{180} = \frac{\pi(3)(35)}{180} = \frac{7\,\pi}{12} cm$$

3. A 10 cm arc has a measure of $\frac{\pi}{6}$ radians. Find the radius of the circle.

$$s = r\theta$$
$$10 = r \cdot \frac{\pi}{6}$$
$$r = \frac{60}{\pi} rad.$$

4. A point on a 7 inch radius wheel travels 5 inches. Find the measure of the arc traveled in radians.

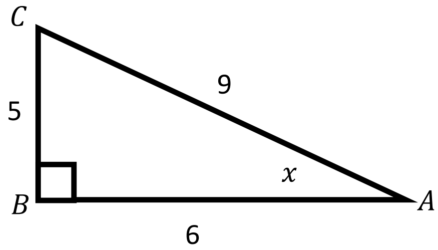
$$s = r\theta$$
$$5 = 7\theta \;\rightarrow\; \theta = \frac{5}{7}$$

5. Find one positive and one negative angle coterminal with an angle of $125°$.

$$360 + 125 = 485°$$
$$-(360 - 125) = -235°$$

Target 7B: In can solve problems by applying the 6 trigonometric functions to the sides of a right triangle.

6. Find the six trigonometric ratios for the triangle

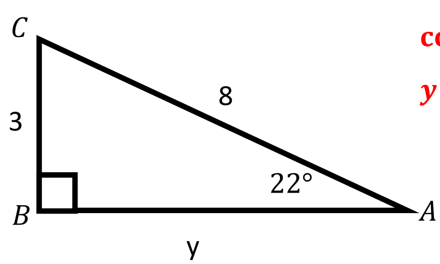


$$\sin x = \frac{5}{9}, \qquad \cos x = \frac{6}{9} = \frac{2}{3}, \qquad \tan x = \frac{5}{6}$$
$$\csc x = \frac{9}{5}, \qquad \sec x = \frac{9}{6} = \frac{3}{2}, \qquad \cot x = \frac{6}{5}$$

7. Find the value of x in the triangle in the previous problem.
   2

$$\sin x = \frac{5}{9}$$
$$\sin^{-1}\left(\frac{5}{9}\right) = 33.7°$$

8. Find the value of y in the triangle below.

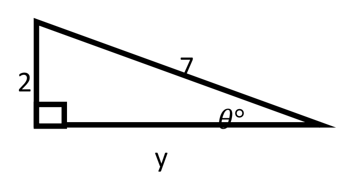


$$\cos 22 = \frac{y}{8}$$
$$y = 8\cos 22 \approx 7.4$$

9. Given that $\csc\theta = \frac{7}{2}$, find the value of the other five trigonometric ratios.
   Draw a reference triangle, and you will find that...

$$hypotenuse = 7\,, \qquad opposite\ leg = 2$$
$$a^2 + 2^2 = 7^2 \quad \rightarrow \quad a = \sqrt{45} = 3\sqrt{5} = adjacent$$

$$\sin\theta = \frac{2}{7}, \cos\theta = \frac{3\sqrt{5}}{7}, \tan\theta = \frac{2}{3\sqrt{5}}, \sec\theta = \frac{7}{3\sqrt{5}}, \cot\theta = \frac{3\sqrt{5}}{2}$$



Target 7C: I can find the values of Sine and Cosine on the 16 point Unit Circle and apply them to solve problems involving all 6 trigonometric functions.

10. Find the *exact values* of the following trigonometric functions using the Unit Circle. Be sure to show work when necessary.

a. $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

b. $\cos\left(\frac{3\pi}{6}\right) = \cos\left(\frac{\pi}{2}\right) = \mathbf{0}$

Remember:

$$\sin\theta = y, \cos\theta = x, \tan\theta = \frac{y}{x},$$
$$\csc\theta = \frac{1}{y}, \sec\theta = \frac{1}{x}, \cot\theta = \frac{x}{y}$$

c. $\tan\left(\frac{3\pi}{4}\right) = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \mathbf{-1}$

d. $\cos(-\frac{\pi}{6}) = \cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

e. $\tan\left(-\frac{2\pi}{3}\right) = \tan\left(\frac{4\pi}{3}\right) = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} =$

$\mathbf{\sqrt{3}}$

f. $\sec\left(\frac{13\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right)$

since $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$,

$\sec\left(\frac{\pi}{6}\right) = \frac{1}{\cos\left(\frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}} = \mathbf{\frac{2\sqrt{3}}{3}}$

g. $\cot\left(\frac{\pi}{3}\right) = \frac{x}{y} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

h. $\sec\left(-\frac{5\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} =$

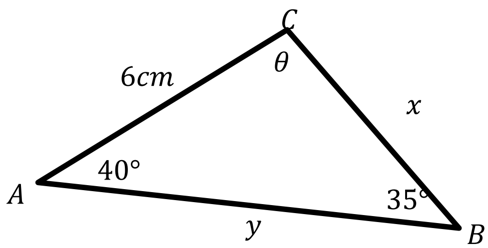
$\frac{1}{\frac{1}{2}} = \mathbf{2}$

i. $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

j. $\sin\left(\frac{15\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Target 7D: I can solve problems using the Law of Sines and the Law of Cosines.

11. Solve the triangle.



$$40 + 35 + \theta = 180$$

$$\boldsymbol{\theta = 105°}$$

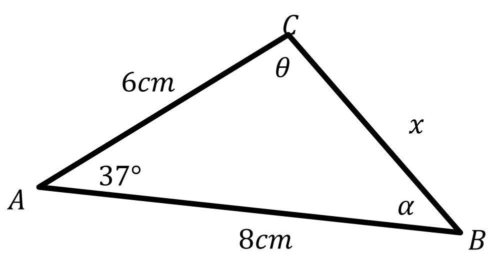
$$\frac{\sin 35}{6} = \frac{\sin 40}{x}$$

$$\boldsymbol{x = \frac{6\sin(40)}{\sin(35)} = 6.72cm}$$

$$\frac{\sin 35}{6} = \frac{\sin 105}{y}$$

$$\boldsymbol{y = 6 \cdot \frac{\sin 105}{\sin 35} = 10.10cm}$$

12. Solve the triangle



$$x^2 = 6^2 + 8^2 - 2(6)(8)\cos 37$$

$$\boldsymbol{x = \sqrt{100 - 96\cos(37)} = 4.8302cm}$$

If you use the law of sine, you get

$$\frac{\sin 37}{4.8302} = \frac{\sin \alpha}{6} \Rightarrow \alpha = \sin^{-1}\left(\frac{6\sin 37}{4.8302}\right) = \boldsymbol{48.38°}$$

$$\boldsymbol{\theta = 180 - 37 - 48.38 = 94.62°}$$

If you use law of cosines, you get:

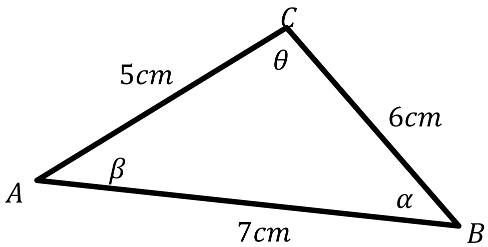
$$8^2 = 6^2 + 4.8302^2 - 2(6)(4.8302)\cos\theta$$

$$\boldsymbol{\theta = \cos^{-1}\left(\frac{8^2 - 6^2 - 4.8302^2}{-2(6)(4.8302)}\right) \approx 94.62°}$$

$$37 + 94.62 + \alpha = 180$$

$$\boldsymbol{\alpha = 48.38°}$$

13. Solve the triangle



$7^2 = 5^2 + 6^2 - 2(5)(6)\cos\theta$

$\boldsymbol{\theta = 78.5°}$

$5^2 = 6^2 + 7^2 - 2(6)(7)\cos\alpha$

$\boldsymbol{\alpha = 44.4°}$

$78.5 + 44.4 + \beta = 180$

$\boldsymbol{\beta = 57.1°}$

14. State if $\triangle ABC$ can be solved if $\angle A = 43.1°, a = 186.2, b = 248.6$. If it can be solved, state if 1, or 2 triangles are possible and how you know.

$$\frac{\sin 43.1}{186.2} = \frac{\sin B}{248.6}$$

$$\sin B = \frac{248.6 \sin 43.1}{186.2}$$

$$B = \sin^{-1}\left(\frac{248.6 \sin 43.1}{186.2}\right) \approx 65.8°$$

Check the sum of $A + B = 43.1 + 65.8 = 108.9° < 180°$, so $B = 65.8°$ is a possible solution. The other possible measure for B is $B = 180 - 65.8 = 114.2°$. Since $43.1 + 114.2 < 180$, this is another possible solutions.

There are two possible triangles because there are two possible values for $\angle B$.

<u>**Applications**</u>

15. A trucks wheels have a diameter of 36 inches. If the wheels are rotating at 500 rpm, find the speed of the car in miles per hour.

$$\frac{500\cancel{rev}}{1\cancel{min.}} \cdot \frac{2\pi\ \cancel{rad}}{1\cancel{rev.}} \cdot \frac{18\cancel{in.}}{1\cancel{rad}} \cdot \frac{60\cancel{min.}}{1hr} \cdot \frac{1\cancel{ft}}{12\cancel{in}} \cdot \frac{1mi}{5280\cancel{ft}} =$$

$$\frac{500 \cdot 2 \cdot \pi \cdot 18 \cdot 60}{12 \cdot 5280} = \frac{375\,\pi}{11} \approx 53.5mph$$

16. Two observers are standing 950 feet apart and looking up at an hot air balloon that is directly between them as shown in the drawing below. One looks up at 75° and the other looks up at 70°.

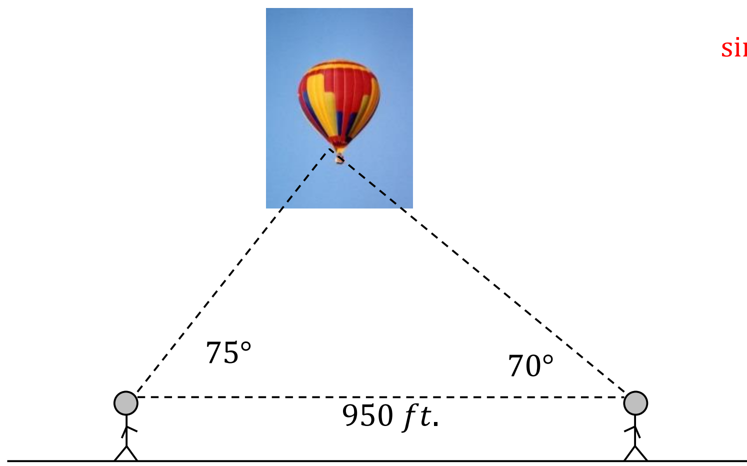
a. Find out how far the balloon is from each observer.

$$\theta + 75 + 70 = 180 \rightarrow \theta = 35°$$

$$\frac{\sin 70}{x} = \frac{\sin 35}{950} \quad \rightarrow \quad x = \frac{950 \sin 70}{\sin 35} \approx \mathbf{1556.4}\ \boldsymbol{ft}.$$

$$\frac{\sin 75}{y} = \frac{\sin 35}{950} \rightarrow y = \frac{950 \sin 75.0}{\sin 35} \approx \mathbf{1599.8}\boldsymbol{ft}.$$

b. Find the height of the balloon.



$$\sin 70 = \frac{h}{1599.8} \rightarrow \boldsymbol{h} = \mathbf{1503}\boldsymbol{ft}.$$

$$\sin 75 = \frac{h}{1556.4} \rightarrow \boldsymbol{h} = \mathbf{1503}\ \boldsymbol{ft}.$$