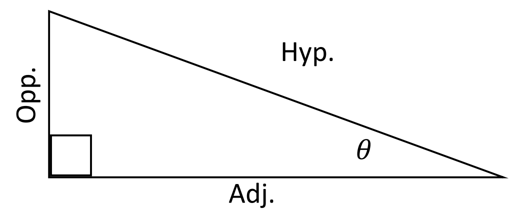
8B.1: Graphs of Tangent & Cotangent

We have seen how the sine and cosine functions give us a nice oscillating wave graph that has many applications for endlessly repeating patterns found in naturally occurring events. Let's now consider the third basic trigonometric function, tangent, and its reciprocal function, cotangent.

From Sine Waves to Tangent Curves

We first need to go back to the right-triangle definition of our trigonometric functions to understand the tangent graph.

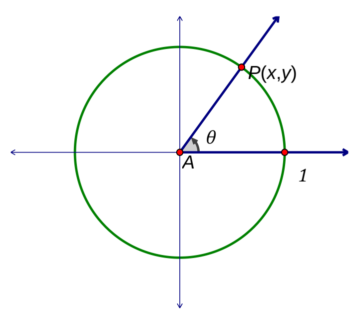
Consider this:



a) Use the right triangle definitions of sine and cosine to write an equation that equates $\sin\theta$, $\cos\theta$, and $\tan\theta$. *Explain/Show why this is true.*

$\tan\theta =$

$\cot\theta =$



b) Now consider a point $P(x, y)$ that is the intersection of an angle of θ radians and the unit circle.
We know that $\cos\theta = x$ and $\sin\theta = y$.
How can we define $\tan\theta$ in terms of x and y ?

$\tan\theta =$

$\cot\theta =$

c) Use the equations above to decide if the tangent and cotangent are positive or negative in each quadrant.

	Quadrant 1	*Quadrant* 2	*Quadrant* 3	*Quadrant* 4
$\tan\theta$				
$\cot\theta$				

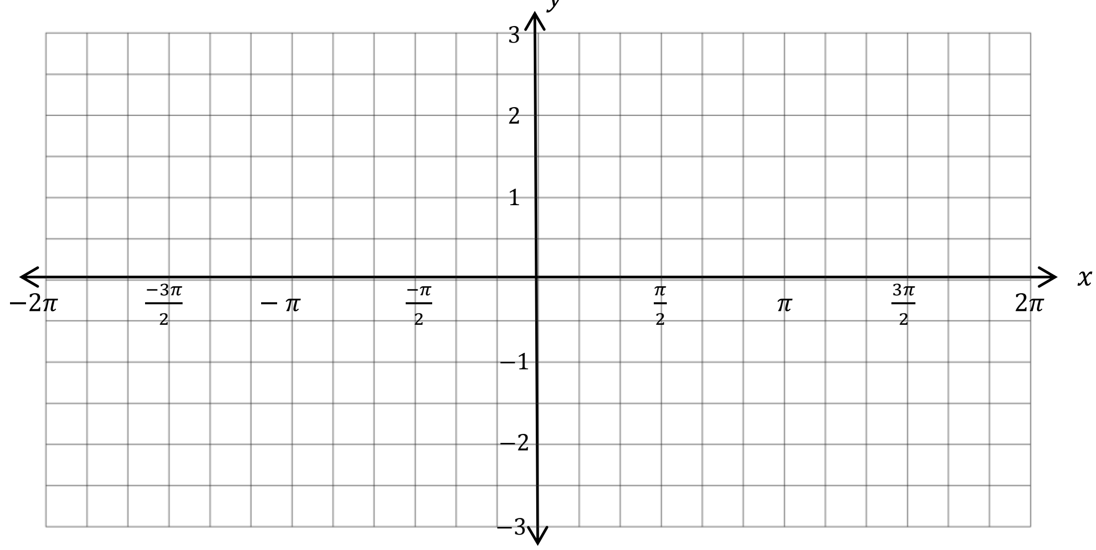
Exploration: Graph by hand

Let's begin by exploring the values of the tangent function in the unit circle like we did with the sine and cosine function. Use the equations above to help.

1. Fill in rows of the following table using our benchmark angles in the unit circle.

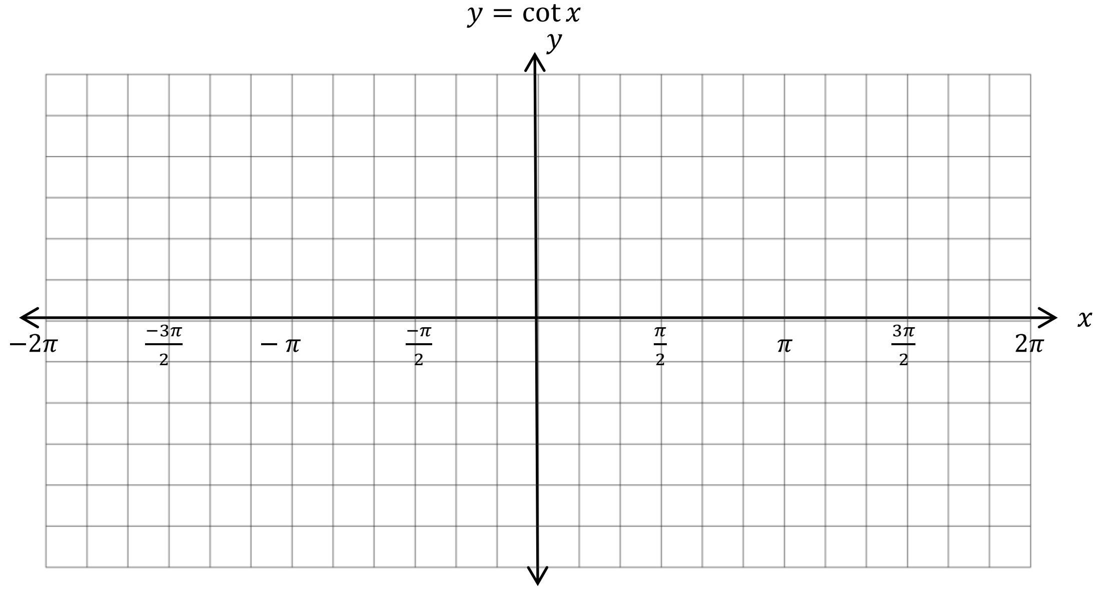
x	π	$3\pi/4$	$\pi/2$	$\pi/4$	0	$-\pi/4$	$-\pi/2$	$-3\pi/4$	$-\pi$
$\tan x$									
$\cot x$									

2. Now plot each ordered pair $(x, \tan x)$ on the graph below for the angles on the interval $(-\pi, \pi)$. $y = \tan x$. Then use what you know about circular functions to complete the graph on $(-2\pi, 2\pi)$



3. Now plot each ordered pair $(x, \cot x)$ on the graph below for the angles on the interval $(-\pi, \pi)$.

$y = \cot x$



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4. Describe any similarities and differences between the graphs of $y = \tan x$ and $y = \cot x$.

5. Use your graph to find the following for $f(x) = \cos(x)$.
(Describe periodic values as "any integer n")

	$y = \tan x$	$y = \cot x$
a. Domain:		
b. Range:		
c. Asymptotes		
d. Zeros		
e. Maximum or Minimum?		
f. Symmetry (odd or even)		
g. Period (horizontal distance required to repeat the curve):		

6. The asymptotes of the *Tangent* curve correspond to the zeros of which other Trig. function?

7. The asymptotes of the *Cotangent* curve correspond to the zeros of which other Trig. function?

Transforming the Tangent and Cotangent Curves

We now want to transform the graphs of $y = \tan x$ and $y = \cot x$. Use your graphing calculator to explore and determine how the constants $a, b, h,$ and k affect the graph of

$$f(x) = a\tan(b(x-h)) + k \text{ and } g(x) = a\cot\big(b(x-h)\big) + k$$

<u>Transformations of tangent and cotangent curves</u>
For the functions $f(x) = a\tan(b(x-h)) + k$ and $g(x) = a\cot\big(b(x-h)\big) + k$, the constants $a, b, h,$ and k do the following:

a: Vertical stretch. With $b = 1$, curves contain points $\left(\frac{\pi}{4} + \pi n, \pm a\right)$ and $\left(\frac{-\pi}{4} + \pi n, \pm a\right)$

b: Horizontal stretch. Period $= \frac{\pi}{|b|}$

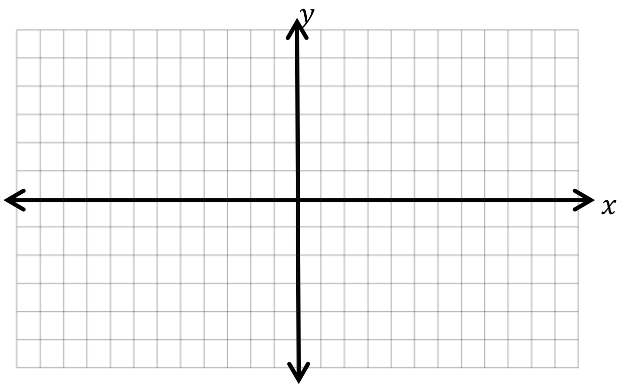
h: Horizontal translation. With $b = 1$,
tangent zeros at $x = h + \pi n$, cotangent zeros at $x = \frac{\pi}{2} + h + \pi n$

k: Vertical translation.

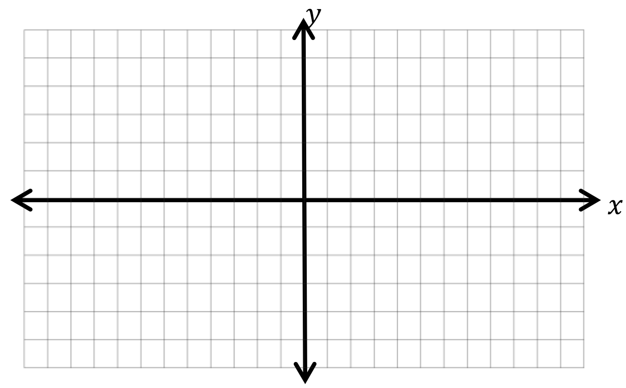
What points should we plot? Begin by (1) plotting the asymptotes, then (2) find the zeros (or their translations), then (3) plot the points that correspond to $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ by moving up and down a units from the zero. Now you can draw in your curves.

Try it! Determine the amplitude and period, and then use transformations to graph the following:
Graph them without your calculator, then check them on your calculator

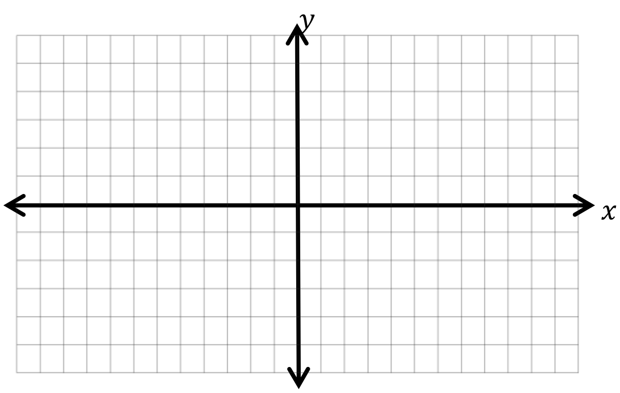
$y = 2\tan(x)$



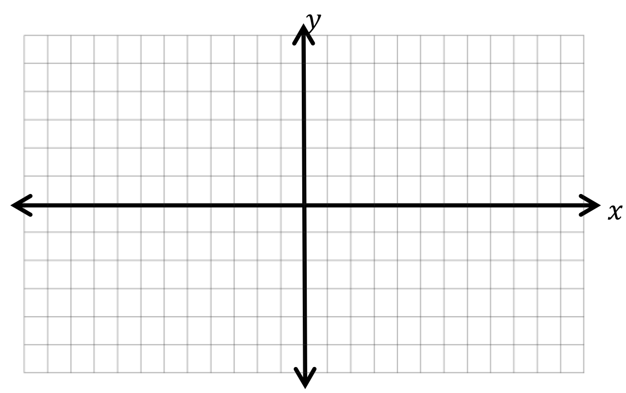
$y = \tan 2x + 1$



$y = -\cot(x)$



$y = \frac{1}{4}\cot\left(x + \frac{\pi}{4}\right)$



Assignment 8B.1-Tangent & Cotangent

Describe how the graph the following curves differs from $y = \tan x$ and $y = \cot x$. Make sure you state the period and the vertical stretch.

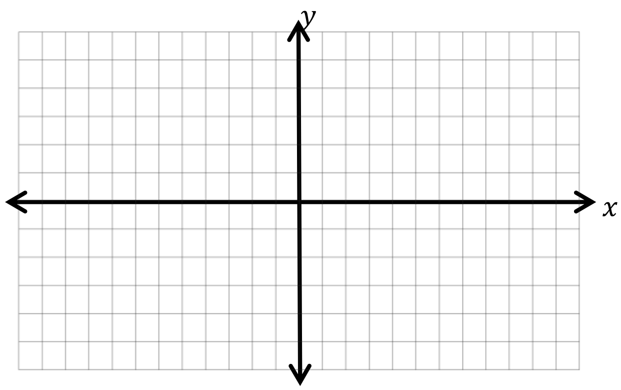
1. $y = 2\tan 3x$

2. $y = -\tan\left(\frac{x}{2}\right) - 5$

3. $y = 5\cot(x + 3) + 12$

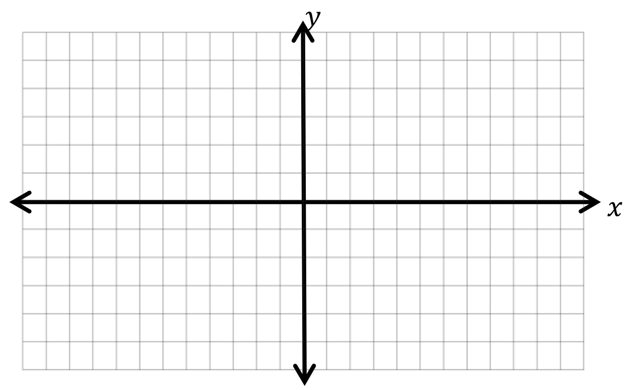
Graph at least two periods for the following functions. State the period of each function and the location of the asymptotes.

4. $y = -2\tan x$



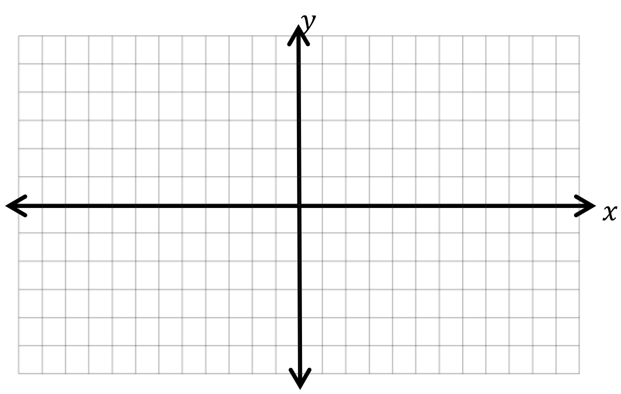
Period=   asymptotes=

5. $y = \frac{1}{2}\tan(x + \pi)$



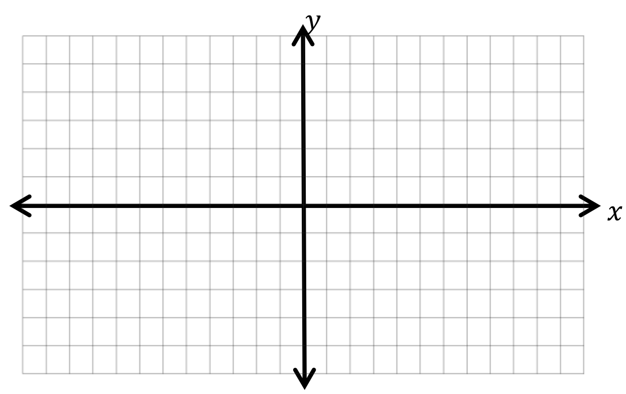
Period=   asymptotes=

6. $y = \cot(2x)$



Period=   asymptotes=

7. $y = \cot(-x)$



Period=   asymptotes=