8C.1: Identifying Composite Trig. Functions

We have now studied polynomial, exponential, logarithmic, rational, and trigonometric functions along with others. We now want to find out what happens when we put two of these functions together. A **composite function** is made of two functions joined by an operation (like addition or multiplication) or function composition (placing one function inside another.) In this lesson, we will explore the nature of some different composite functions.

Identifying and Analyzing Periodic Functions

Explore

Graph these functions on your calculator, sketch the graph, and decide which ones have periodic behavior.

a) $y = \sin x + x^2$

b) $y = \sin^2 x$
(*note: this is shorthand for* $y = (\sin x)^2$)

c) $y = x^2 \sin x$

d) $y = \sin(x^2)$

Example *Verifying Periodicity Algebraically*

Let's consider the function $y = \sin^3 x$ and verify that it is periodic

a) Graph $y = \sin^3 x$ on your calculator.
b) Find the following from the graph:

domain = range = Period =

c) Use the period to show that $f(x + 2\pi) = f(x)$

Example. *Analyzing Non-negative Periodic Functions*

Let's consider the absolute value of a tangent function. Find the domain, range, and period of each function, then sketch the graph.

a) $f(x) = |\tan x\ |$

b) $g(x) = |\sin x|$

Sinusoids from Sums and Products of functions

Recall that a sinusoid is a function that can be written in the form

$$y = a\sin\big(b(x-h)\big) + k.$$

Looking at a graph, a sinusoid is simple a transformed sine wave.

Graph the following functions on your calculator, sketch the graph, and decide if they are periodic or not, and if it is a sinusoid.

a) $y = x + \sin x$

b) $y = x \sin x$

c) $y = 3\sin x + 2\cos x$

d) $y = \sin 3x + \cos 2x$

e) $y = \sin\left(\frac{x}{5} + 2\right) + \cos\left(\frac{x}{5} + 3\right)$

Key: A sum (or difference) of two sinusoid functions is a sinusoid if they have the same _____________________________

Example. Is this a sinusoid: $y = 23\sin\left(\frac{2}{3}(x+4)\right) + 34\cos\left(\frac{2}{3}(x-40)\right) - 45\sin\left(\frac{2x-5}{3}\right)$?

Damped Oscillation

We have seen that many natural events like bouncing springs and swinging pendulums can be modeled by a sinusoid. However in the real world, the values of these functions slowly decay. To model this, we use a damped oscillation.

For a function

$$f(x) = g(x)\sin x\,, \qquad or \quad f(x) = g(x)\cos x$$

$g(x)$ is the *dampening factor*

Explore Graph the following on your calculator, sketch the graph and describe how the two graphs, and determine what the dampening factor is. Note: you may want to use a ZOOM Fit.

a) $y = \frac{1}{x}\cos x$

b) $y = x\sin 5x$

c) $y = e^{-.005x}\cos x$