9A.1: Intro. To Trigonometric Identities

We have seen that trigonometric ratios are useful for modeling real life events that are periodic. Since trigonometric ratios appear often in applications, we need to develop some techniques for solving trigonometric equations. The inverse functions are useful for solving simple trig. equations such as $\sin x = .35$. However, we need to use a more powerful tool to solve more complex equations, the tools we often need are ***trigonometric identities***

Exploring Identities

An *identity* is a mathematical relationship that states that two quantities (that may appear to be different) are equal.

Consider These:

Which of the following are true identities? (Hint: you may need to factor some of the quadratics.)

a. $\frac{x^2+5x+6}{x+2} = x + 3$

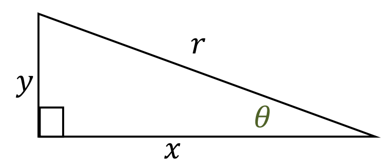
b. $\sqrt{a^2 + b^2} = a + b$

c. $e^m e^n = e^{mn}$

d. $\sqrt{x^2 + 4x + 4} = |x + 2|$

Reciprocal Trigonometric Identities

Some identities come from their definition.



Reviewing The Six Functions

Use the sides of the triangle to the right to write ratios for each of the trigonometric ratios (we will refer back to these as we prove and verify identities)

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

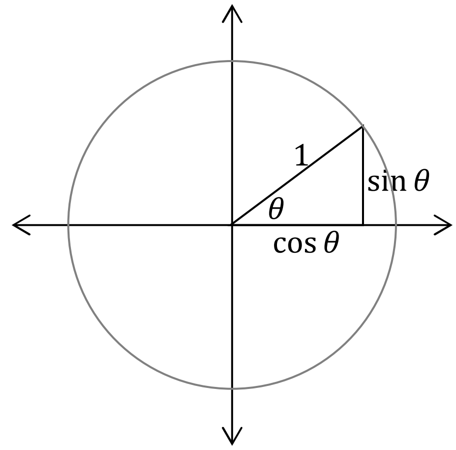
$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

Relating Functions

Use the definitions of these ratios to complete each of the identities below with the appropriate trigonometric ratio.

$$\sin\theta = \frac{1}{\quad} \qquad \cos\theta = \frac{1}{\quad} \qquad \tan\theta = \frac{1}{\quad}$$

$$\csc\theta = \frac{1}{\quad} \qquad \sec\theta = \frac{1}{\quad} \qquad \cot\theta = \frac{1}{\quad}$$



Pythagorean Identities

We have learned that the length of the horizontal and vertical legs of the reference triangle for any angle θ determine the cosine and sine of the angle.

Consider this:

1. Use Pythagorean Theorem to write an equation relating the sides of the reference triangle to the right.

2. Now rewrite this equation in #1 by dividing both sides by $\sin^2 x$ and simplify it to eliminate any fractions. (Hint: use the Reciprocal identities)

3. Repeat this process by dividing the equation in #1 by dividing both sides by $\cos^2 x$ and simplify it to eliminate any fractions.

Verifying on a Graph

1. Graph $y = \sin^2 x$ and $y = \cos^2 x$
2. Describe how these two curves relate to each other as you move left to right?

3. Now graph $y = \sin^2 x + \cos^2 x$. Describe this new graph.

4. Now graph $y = \cot^2 x$ and $y = \csc^2 x$. How are these graphs related?

5. Finally, graph $y = \tan^2 x$ and $y = \sec^2 x$. How are these graphs related?

Useful Pythagorean Forms

Pythagorean Identities *The main Pythagorean identities are written below. Rewrite them to fill in the identities below.*		
$\mathbf{\sin^2 x + \cos^2 x = 1}$	$\mathbf{1 + \cot^2 x = \csc^2 x}$	$\mathbf{\tan^2 x + 1 = \sec^2 x}$
$\sin^2 x =$	$\cot^2 x =$	$\tan^2 x =$
$\cos^2 x =$	$1 =$	$1 =$

Using Identities to Simplify

These identities are useful when we need to replace one trigonometric ratio with a form of a different ratio.

Try These

Use the reciprocal and Pythagorean identities to rewrite and simplify the expressions as much as possible.

$\csc x \cdot \tan x$

$\sin x \cdot \sec x \cdot \cot x$

$(\sin^2 x + \cos^2 x)(\cot^2 x + 1)$

$$\frac{1 + \tan^2 x}{\csc^2 x}$$

$(1 - \cos^2 x)(\csc^2 x - 1)$

$$\frac{\sin^2 x + \cos^2 x + \tan^2 x}{\sec x}$$

$(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$

Useful Techniques

1. Direct Substitution (As we did above)
2. Change the identity to one involving only sines and cosines
3. Multiply or simplify parts of the expression to make a recognizable power of a trig. Function.
   Example:Simplify $(\sec x + 1)(\sec x - 1) \cdot \cot^2 x$

4. Factoring
   Example: *Simplify* $\cos^3 x + \cos x \sin^2 x$

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Simplify each expression using the trigonometric identities and substitution. Show your steps

1. $\cos x \cdot \tan x$

2. $\cos x \cdot \csc x$

3. $\csc x - \csc x \cdot \cos^2 x$

4. $\frac{1-\cos^2 x}{\sin x}$

5. $\frac{\sin^2 x+\tan^2 x+\cos^2 x}{\sec x}$

6. $\sin x + \sin x \tan^2 x$ *(Try factoring)*

7. $\frac{\sec^2 x-\tan^2 x}{\cos^2 x+\sin^2 x}$

8. *(Hint: Write in terms of sine and cosine. Then make a compound fraction)*
   $\frac{1+\tan x}{1+\cot x}$