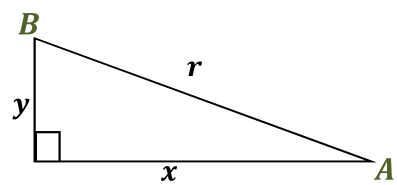
Pre-Calculus Name: Date: Period:

9A.2: More Identities and Trig. Equations

We have seen how to use some fundamental identities to simplify expressions. In this lesson, we will continue to explore two more families of trigonometric identities.

Cofunction Identities

Our next set of identities takes us back to the definitions of the 6 trigonometric functions and how they apply to acute angles in a right triangle.



Consider This

1. How do $m\angle A$ and $m\angle B$ relate to each other in the triangle to the right?

2. Use the triangle to the right to complete second row in the table below.

$\sin A = \frac{y}{r}$	$\cos A = \frac{x}{r}$	$\tan A = \frac{y}{x}$	$\csc A = \frac{r}{y}$	$\sec A = \frac{r}{x}$	$\cot A = \frac{x}{y}$
$\sin B =$	$\cos B =$	$\tan B =$	$\csc B =$	$\sec B =$	$\cot B =$

3. Draw lines to connect equivalent expressions.
4. If $m\angle A = \theta$, then we know that $m\angle B = \frac{\pi}{2} - \theta$. In #3 above, we should have seen that $\sin B = \cos A$, so we can write $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$.
   Use this substitution to complete the *cofunction identities* below:
   Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) =$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \qquad \cot\left(\frac{\pi}{2} - \theta\right) =$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \qquad \csc\left(\frac{\pi}{2} - \theta\right) =$$

Odd-Even Identities

We have seen that some functions are even functions and some functions are odd functions. If a function is an even function , like $f(x) = x^2$, then $f(-x) = f(x)$. If a function is odd, like $f(x) = x^3$, then $f(-x) = -f(x)$.

Explore

Use the graphs of each function to decide if it is even or odd, and complete the identities

Odd- Even Identities

$\sin(-x) = -\sin x$ $\qquad\qquad$ $\csc(-x) =$

$\cos(-x) =$ $\qquad\qquad$ $\sec(-x) =$

$\tan(-x) =$ $\qquad\qquad$ $\cot(-x) =$

Solving Equations

Now that we are familiar with the fundamental trigonometric identities, we can use them to solve trigonometric equations. When solving a trig. equation, you can think about these three basic steps:

1. **Simplify** the trig. expressions on each side of the equation using identities to get a simple function(s) of x
2. **Isolate** the trig. expressions to get constants on one side.
   (You may need to factor first and make 2 equations)
3. **Solve** using inverse trig. functions.

Example Find all solutions to the equations on the interval $[0,2\pi)$

a) $2\cos^2 x = 1$

b) $2\sin\left(\frac{\pi}{5}x\right) = -\sqrt{3}$

c) $\sin^3 x + \sin x \cos^2 x + 2 = 1$

d) $\sin(-x) = \frac{1}{2}$

e) $2\cos x \sin x - \cos x = 0$ (*Hint: try factoring first*)

f) $2\cos^2 x - \sin x = 1$ (*Hint: Write in terms of sine first, then use quadratic factoring*)

Assignment 9A.2: More Fund. Identities

Find all the exact solutions to the equation in the interval $[0,2\pi)$ without using a calculator.

1. $\tan^2 x = 3$
2. $2\sin^2 x = 1$
3. $\sqrt{2}\tan x \cos x - \tan x = 0$
4. $\tan x \sin^2 x = \tan x$
5. $\sin x \tan^2 x - \sin x = 0$

Find all the exact solutions to the equation in the interval $[0,2\pi)$ using quadratic factoring. (No calculator needed).

6. $4\cos^2 x - 4\cos x + 1 = 0$
7. $2\sin^2 x + 3\sin x + 1 = 0$
8. $\sin^2 t = 2\sin t$
9. $3\sin y = 2\cos^2 y$