Pre-Calculus

Unit 9 Toolkit – Analytic Trigonometry

This toolkit is a summary of some of the key topics you will need to master in this unit.

9A: Fundamental Trigonometric Identities

Learning Target: I can use and apply fundamental trigonometric identities.

Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$	$\cos x = \dfrac{1}{\sec x}$	$\tan x = \dfrac{1}{\cot x}$
$\csc x = \dfrac{1}{\sin x}$	$\sec x = \dfrac{1}{\cos x}$	$\cot x = \dfrac{1}{\tan x}$

Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x}$
$\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$	$1 + \cot^2 x = \csc^2 x$	$\tan^2 x + 1 = \sec^2 x$

<u>Useful Techniques for simplifying Trigonometric Expressions</u>

1. Direct Substitution (As we did above)
2. Change the identity to one involving only sines and cosine.
3. Multiply or simplify parts of the expression to make a recognizable power of a trig. Function.
4. Factoring.

Co-Function Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$	$\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$	$\tan\left(\dfrac{\pi}{2} - x\right) = \cot x$
$\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$	$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$	$\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

Even-Odd Identities

$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$	$\tan(-x) = -\tan x$
$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$	$\cot(-x) = -\cot x$

Solving Equations

Now that we are familiar with the fundamental trigonometric identities, we can use them to solve trigonometric equations. When solving a trig. equation, you can think about these three basic steps:

1. **Simplify** the trig. expressions on each side of the equation using identities to get a simple function(s) of x
2. **Isolate** the trig. expressions to get constants on one side.
   (You may need to factor first and make 2 equations)
3. **Solve** using inverse trig. functions.

9B: Proving Trigonometric Identities

Learning Target: I can prove trigonometric identities.

Key Concept: Each expression in a proof should be equivalent to the original expression.

General Strategies 1

1. The proof begins with the expression on one side of the identity
2. The proof ends with the expression on the other side
3. The proof in between consists of showing a sequence of expressions, each one *easily seen* to be equivalent to its preceding expression.

General Strategies II

1. Begin with the more complicated expression and work toward the less complicated expression.
2. If no other move suggests itself, convert the entire expression into one involving sines and cosines.
3. Combine fractions by combining them over a common denominator.

General Strategies III

1. Use the algebraic identity $(a+b)(a-b)=a^2-b^2$ to set up applications of the Pythagorean identities.
2. Always be mindful of the "target" expression, and favor manipulations that bring you closer to your goal.

9C: Angle Sum and Difference identities

Learning Target: I can use and apply sum and difference identities.

Sine and Cosine Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

*Note: Notice that the sign in the RHS of the **cosine** identity are opposite of the sign of the LHS. However, the sign is the same for both RHS and LHS of the sine angle-sum identity.*

Tangent Angle sum Identity

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

9D: Multiple Angle Identities

Learning Target: I can use and apply multiple angle identities.

Remember…

- $\sin(2u) = \sin(u+u)$
  $= \sin u \cos u + \sin u \cos u$
  $= 2\sin u \cos u$

- $\cos(2u) = \cos(u+u)$
  $= \cos u \cos u - \sin u \sin u$
  $= \cos^2 u - \sin^2 u$

Double-Angle Identities

$$\sin(2u) = 2\sin u \cos u$$

$$\cos(2u) = \begin{cases} \cos^2 u - \sin^2 u \\ 2\cos^2 u - 1 \\ 1 - 2\sin^2 u \end{cases}$$

$$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$$

Half-Angle Identities

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \begin{cases} \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}} \\ \dfrac{1-\cos u}{\sin u} \\ \dfrac{\sin u}{1+\cos u} \end{cases}$$

Power-Reducing Identities

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$