10A: Vectors in the Plane

While some quantities can be measured with a single real number, many quantities requires two numbers such as force which has both a magnitude and a direction. For this type of quantity, we need to use a ***vector*** to describe both important values.
In this lesson, we will learn how to describe, compute, and apply vectors to solve problems.

What's a Vector?

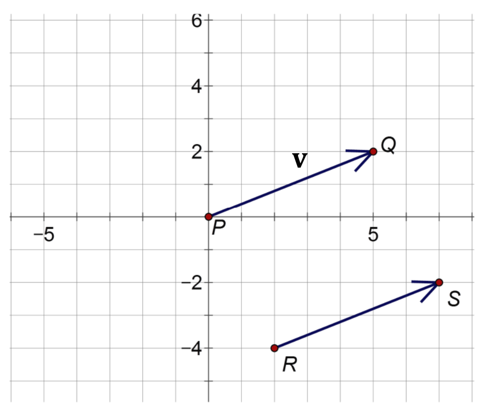
A quantity that can be described with a single real number is called a **scalar**.

However, a **vector** is quantity with ____________________ and ____________________.

Graphically, a vector is represented by a *directed line segment* with a specific length and direction. The key to graphing a vector is that the

$$\boldsymbol{length} = \boldsymbol{magnitude}$$

On the coordinate plane to the right, we have two vectors graphed. The first vector from $P(0,0)$ to $Q(5,2)$ is called $\overrightarrow{PQ}$, and the magnitude is labeled as $|\overrightarrow{PQ}|$.



Some key features of vectors are

- **Initial Point:**
- **Terminal Point:**
- **Standard Position:**
- **Equivalent vectors**

Try This: Find the magnitude of each of the vectors above.

Component Form of a Vector

To describe vectors in standard position as $\mathbf{v} = \overrightarrow{PQ}$, we will use the terminal point $(5,2)$ to describe the vector in **component form** of $\mathbf{v}$ as $\mathbf{v} = \langle 5,2\rangle$.

Component Form of Vector
The component form of a standard position vector with terminal point (v_1, v_2) is
$$\mathbf{v} = \langle v_1, v_2\rangle$$

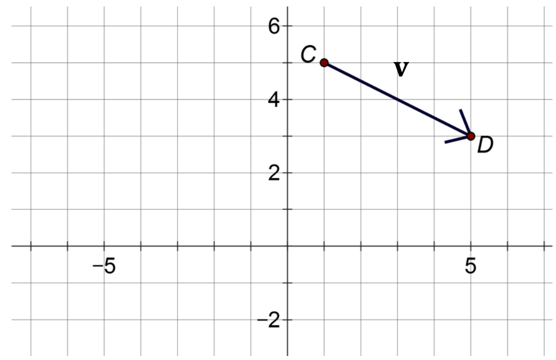
Try This Plot and label these vectors above.

a) $\mathbf{w} = \langle -3,2\rangle$ *b)* $\mathbf{r} = \langle 3,-2\rangle$

*These vectors are called **opposite vectors.***

What if a vector is not in standard position? We can still describe this vector in component form by considering the equivalent vector in standard position.

Consider this
Find the component form and the magnitude of vector $\mathbf{v} = \overrightarrow{CD}$ *in the graph to the right.*



General Component form and Magnitude:
The vector $\mathbf{v} = \overrightarrow{PQ}$ with $P(x_1, y_1)$ and $Q(x_2, y_2)$ has component form

$$\mathbf{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

and **magnitude** of

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note: One special vector is the **zero vector** written as $\langle 0,0 \rangle$ which has no direction and no magnitude.

Vector Operations

When working with vectors (or "Vector Spaces") there are two operations:
Scalar Multiplication, and Vector addition.

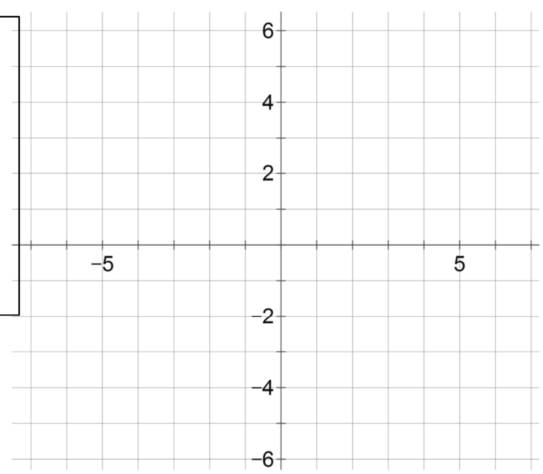
Scalar Multiplication: For scalar a and vector $\mathbf{v} = \langle v_1, v_2 \rangle$,

$$a\mathbf{v} = \langle a \cdot v_1, a \cdot v_2 \rangle$$

Vector Addition: For vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$,

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

Note: *The sum* $\mathbf{u} + \mathbf{v}$ is called the ***resultant*** vector.



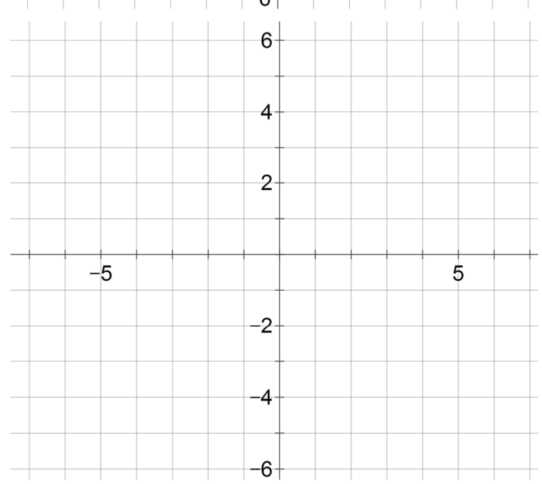
Example Let $\mathbf{u} = \langle -2,3 \rangle$ and $\mathbf{v} = \langle 2,1 \rangle$

a) Plot $\mathbf{u} = \langle -2,3 \rangle$ and $\mathbf{v} = \langle 2,1 \rangle$ on the top axes to the right.
b) Find $2\mathbf{u}$ and plot it on the top axes.

c) Find $-\mathbf{v}$ and plot it on the top axes

d) Plot $\mathbf{u}$ and $\mathbf{v}$ on the lower axes.
   Find $\mathbf{u} + \mathbf{v}$ and plot this vector on the lower axes.

e) Find $\mathbf{v} + (-1)\mathbf{v}$



Unit Vectors

A **Unit vector** is a vector that has a length of 1. If **v** is not the zero vector, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\mathbf{1}}{|\mathbf{v}|}\mathbf{v}$$

is a **unit vector in the direction of v.**

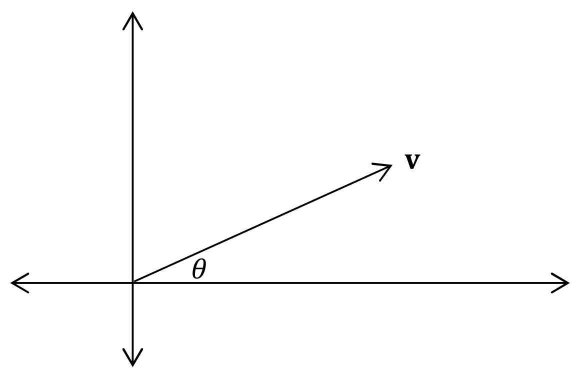
Try These Find a unit vector in the direction of $\mathbf{v} = \langle -4,5 \rangle$ and verify that it has a length of 1.

The **Standard unit vectors** are the vectors $\mathbf{i} = \langle 1,0 \rangle$ and $\mathbf{j} = \langle 0.1 \rangle$. Any vector can be written in terms of this vector, this is called a ***linear combination*** of the vectors **i** and **j**.

Consider This Let **v** is a vector between the initial point $A(2,0)$ and terminal point $B(-1,3)$.
Sketch a graph of the vector, write **v** in terms of **i** and **j** vectors, then write it in component form.

Direction Angles

Suppose we know vector **v** and the angle θ with the x axis. We can define this state as a direction vector.



Think about it: Find the horizontal and vertical components of **v** in terms of **v** and θ.

In terms of the standard unit vectors:

$$\mathbf{v} = (|\mathbf{v}|\cos\theta)\mathbf{i} + (|\mathbf{v}|\sin\theta)\mathbf{j}$$

Finally, find the unit vector $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$

Try This: Find the magnitude and directional angle of the vector $\langle 3,2 \rangle$.

Assignment 10A: Vectors in a Plane

Let **v** is a vector between the initial point A and terminal point B with the given coordinates. *Sketch* a graph of the vector, write **v** in terms of **i** and **j** vectors, then write it in component form.

1. $A(1,2),\ B(-1,5)$

2. $A(-2,-3),\ B(0,2)$

3. $A(-7,10),\ B(6,-12)$

Let $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$, and $\mathbf{w} = -4\mathbf{i}$. Find the following

4. $\mathbf{u} + \mathbf{3v}$

5. $2\mathbf{u} - (4\mathbf{v} - \mathbf{w})$

For each of the following vectors, find the magnitude $|\mathbf{u}|$ (exactly) and the direction angle (to the nearest tenth of a degree).

6. $\mathbf{u} = \langle 5,12 \rangle$

7. $\mathbf{u} = \langle -4,-10 \rangle$

8. The magnitude and direction of two forces acting on an object are 75 pounds at 75° and 45 pounds at 130°. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.