

Assignment 10B.1-Dot Products

Let $\mathbf{u} = \langle 3, 6 \rangle$, $\mathbf{v} = \langle -4, 3 \rangle$, and $\mathbf{w} = \langle 3, 4 \rangle$

1. Find the following dot products

a. $\mathbf{u} \cdot \mathbf{v}$

$$\langle 3, 6 \rangle \cdot \langle -4, 3 \rangle = -12 + 18 = 6$$

b. $\mathbf{u} \cdot \mathbf{w}$

$$\langle 3, 6 \rangle \cdot \langle 3, 4 \rangle = 9 + 24 = 33$$

c. $\mathbf{v} \cdot \mathbf{w}$

$$\langle -4, 3 \rangle \cdot \langle 3, 4 \rangle = -12 + 12 = 0$$

2. Find the angle between each pair of vectors:

$$|\mathbf{u}| = \sqrt{3^2 + 6^2} = \sqrt{45}, \quad |\mathbf{v}| = \sqrt{(-4)^2 + 3^2} = 5, \quad |\mathbf{w}| = \sqrt{3^2 + 4^2} = 5$$

a. \mathbf{u} , and \mathbf{v}

$$\cos^{-1}\left(\frac{6}{\sqrt{45} \cdot 5}\right) = 79.7^\circ$$

b. \mathbf{u} , and \mathbf{w}

$$\cos^{-1}\left(\frac{33}{\sqrt{45} \cdot 5}\right) = 10.3^\circ$$

c. \mathbf{v} , and \mathbf{w}

$$\cos^{-1}\left(\frac{0}{5 \cdot 4}\right) = 90^\circ$$

3. Which vectors pairs above are orthogonal? Explain.

\mathbf{v} and \mathbf{w} are orthogonal because their dot product is zero which makes the angle between them 90°

4. Let $\mathbf{r} = \langle x, -2 \rangle$. Find the value of x that will make \mathbf{u} and \mathbf{r} orthogonal.

$$\mathbf{u} \cdot \mathbf{r} = \langle 3, 6 \rangle \cdot \langle x, -2 \rangle = 3x - 12 = 0$$

$$x = \frac{12}{3} = 4$$

5. Let $\mathbf{s} = \langle -5, y \rangle$. Find the value of x that will make \mathbf{v} and \mathbf{s} orthogonal.

$$\mathbf{v} \cdot \mathbf{s} = \langle -4, 3 \rangle \cdot \langle -5, y \rangle = 20 + 3y = 0$$

$$y = -\frac{20}{3}$$

6. Find $\mathbf{u} \cdot \mathbf{v}$ satisfying the given conditions where θ is the angle between \mathbf{u} and \mathbf{v} .

$$\theta = 100^\circ, |\mathbf{u}| = 6, |\mathbf{v}| = 9$$

$$\cos 100 = \frac{(\mathbf{u} \cdot \mathbf{v})}{6 \cdot 9}$$

$$\mathbf{u} \cdot \mathbf{v} = 54 \cos 100 \approx -9.38$$