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10B.2: Projections And Applications

In the last lesson, we were introduced to the vector *dot product* defined as $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$. We learned how this can be applied to find the angle between two vectors and specifically to know if two vectors are orthogonal (i.e. perpendicular).

Projection of one vector onto another

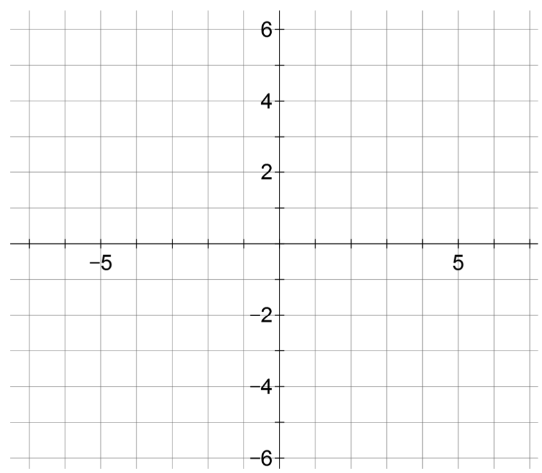
In the application of vectors, it is very useful to find the effect of a given vector in a certain direction.

Projection of u onto v:

For nonzero vectors **u** and **v**, the projection of **u** onto **v** is

$$proj_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right) \cdot \mathbf{v}$$

Explore: For each of the following, graph the vectors, find the projection of the first vector onto the second, and graph the projection vector.

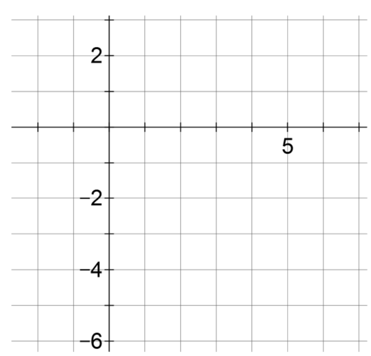


a) $\mathbf{u} = \langle 3,4 \rangle$, $\mathbf{v} = \langle 6,0 \rangle$

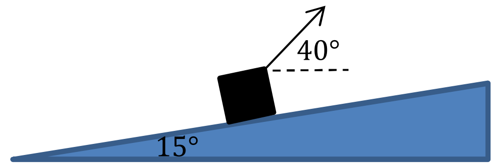
b) $\mathbf{p} = (-2\mathbf{i} + 4\mathbf{j})$, $\mathbf{q} = (-6\mathbf{i} + 1\mathbf{j})$

Consider this: How does the projection vector relate to the first vector? To better see this, connect the terminal points of the projection vector and the vector being projected.

Try it: Find the projection of $\mathbf{u} = \langle 4,2 \rangle$ onto $\mathbf{v} = \langle 6,-6 \rangle$. Then write **u** as the sum of two orthogonal vectors, one of which is the $proj_{\mathbf{v}}\mathbf{u}$.



Example A 50 pound force **F** that makes an angle of 40° with the horizontal is pulling a box up the plane. The inclined plane makes a 15° angle with the horizontal. What is the magnitude of the effective force pulling the box up the plane?



Work

In physics, work is the amount of force applied over a certain distance. We measure work as a product of a magnitude force f and the distance d in the direction of the force. In terms of vectors, we get the definition below.

> **Work:**
> If **F** is a constant force whose direction is at an angle of θ with the direction of $\overrightarrow{AB}$, then the **work** W done by **F** in moving an object from point A to point B is
> $$W = |\mathbf{F}|\left|\overrightarrow{AB}\right| \cos\theta$$
> or
> $$W = \mathbf{F} \cdot \overrightarrow{AB}$$

Try This: Consider the box in the example above that is being pulled by a 50 pound force **F** that makes an angle of 25° with an inclined plane. Find the work done on the box is pulled along the vector $\overrightarrow{\mathrm{AB}} = -6.2\mathbf{i} + 23.2\mathbf{j}$ from point A to point B, find the work done by the force .

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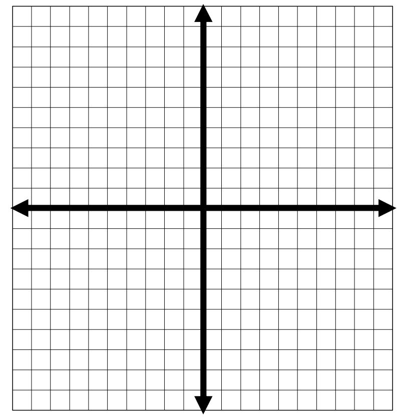
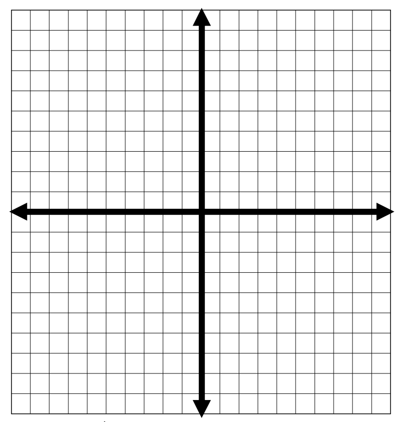
Assignment 10B.2: Projections



1. Let $\mathbf{u} = \langle 6,5 \rangle$, and $\mathbf{v} = \langle 10,1 \rangle$
   a. Find $\mathbf{p} = proj_{\mathbf{v}}\mathbf{u}$

   b. Graph $\mathbf{u}, \mathbf{v}, and\ \mathbf{p}$ on the same axes.



2. Let $\mathbf{u} = \langle -5,5 \rangle$, and $\mathbf{v} = \langle 6,2 \rangle$
   a. Find $\mathbf{p} = proj_{\mathbf{v}}\mathbf{u}$

   b. Graph $\mathbf{u}, \mathbf{v}, and\ \mathbf{p}$ on the same axes.

3. Find the work done by the force vector $\mathbf{F}$ in the direction of the vector $\overrightarrow{AB}$
   a. $\mathbf{F} = \langle 20,4 \rangle,\ |\overrightarrow{AB}| = 30\ ft.,\theta = 40°$

   b. $\mathbf{F} = \langle 20,4 \rangle,\ \overrightarrow{AB} = \langle -5,13 \rangle$

4. A force $\vec{\mathbf{F}}$ is acting on a rope to pull a box up an inclined ramp as shown in the diagram. Find (i) the component form of the force vector, and (ii) the amount of work done on the box if it is pulled up the ramp <u>10 feet</u> with the information given.
   a. $\alpha = 45°, \theta = 15°,\ |\mathbf{F}| = 15\ pounds$

   b. $\alpha = 50°, \theta = 10°,\ |\mathbf{F}| = 30\ pounds$