10C: Parametric Equations

When a non-vertical projectile flies through the air, it's undergoes change in both the horizontal and vertical change depending on the independent variable time. When a business sells a product, the expenses and income are both affected by the independent variable of the number of sales. Both of these situations can be described nicely using two separate equations that describe the variable's change in terms of a **parameter** (e.g. time or number of sales).

When a situation is defined by two equations that depend on a third variable (called the parameter), they are called **parametric equations**.

Definition: ***Parametric Curve***

The set of all ordered pairs (x, y) defined by the equations :

$$x = f(t),\ y = g(t)$$

For some interval $I = [a, b]$ of the parameter t.

Try this:

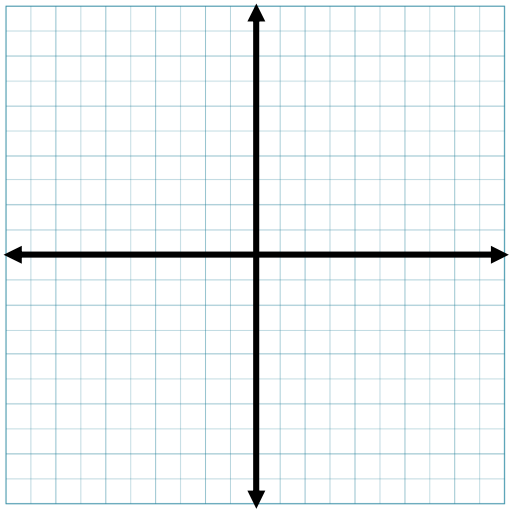
Fill in the table for the given parametric equations and draw the graph for the given interval

$$x(t) = 2t, \qquad y(t) = t^2 - 3$$

t	-3	-2	-1	0	1	2	3
$x(t)$							
$y(t)$							

Graphing Parametric curves using your graphing calculator:

1. Change mode to Parametric.
2. Enter equations for both x= and y=
3. Use [Window] to set appropriate [Tmin] and [Tmax] for your t-interval



Now use your graphing calculator to graph the functions

$$x(t) = 2t, \qquad y(t) = t^2 - 3$$

on the give interval, then sketch the graph.

a) I=[-2,1],
b) I=[-1,4],
c) I=[-4,4]

Eliminating the Parameter

Sometimes it is possible to obtain a rectangular equation in terms of x and y by "eliminating the parameter". This can allow us to identify the graph and perform familiar operations on the rectangular equation.

To eliminate a parameter:

1. Solve one equation for t, and
2. Substitute into the other equation.

Try This: Eliminate the parameter and identify the graph of the parametric curve.

a) $x = 3 - 2t, \quad y = t + 4, \quad -\infty < t < \infty$

b) $x = 2t, \quad y = t^2 - 3, \quad -\infty < t < \infty$

c) $x = 4\cos t\,, \quad y = 4\sin t$ (*Hint: Square both equations and add!*)

Projectile at an angle

If we ignore wind resistance, the vector of a projectile launched at speed v_0 at angle θ can be represented as

$$\mathbf{v} = \langle v_0 \cos\theta\,, v_0 \sin\theta \rangle.$$

The position of the object can be modeled by the parametric equations

$$x = (v_0 \cos\theta)t, \qquad y = -16t^2 + (v_0 \sin\theta)t + y_0$$

where y_0 is the initial height.

Example

Kevin hits a baseball at 3 ft above the ground with an initial speed of 150 ft/sec at an angle of 18° with the horizontal. Will the ball clear a 20-ft wall that is 400 ft away?

Assignment 10C: Parametric Equations

Graph the parametric curves on your calculator and sketch the result.

1. $x = 4\cos^3 t\,,\ y = 2\sin^3 t$

2. $x = 3\cos t\,,\ y = \sin 2t$

3. $x = 2\cos t + 2\cos^2 t\,,\ y = 2\sin t + \sin 2t$

4. $\sin t - t\cos t\,,\ y = \cos t + \sin t$

Eliminate the parameter and identify the graph of the parametric curve. Then graph the parametric curve on your calculator to verify your answer.

5. $x = 2 - 3t,\ y = 5 + t$

6. $x = t,\ y = t^2 - 3$

7. $x = t^2,\ y = t + 1$ (*hint: solve for x in terms of y)*

8. $x = 4\sin t\,,\ y = 4\cos t$

9. A rocket was shot in the air at an angle of 60° with the ground at $200 ft/sec$ with an initial height of 2 feet. What is the horizontal distance it will travel in 5 seconds? What will its height be in 5 seconds?