

1C: Piecewise Functions

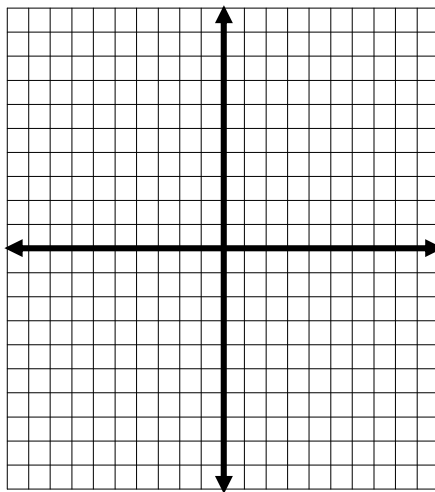
A function is used to describe a relationship between two or more variables. However, sometimes this relationship changes for different input values. In this lesson, we will learn about a type of function called a **piecewise function** which is a way to combine parts of different functions into one function.

Explore

- Graph these functions on separate graphs
 - $y = -\frac{2}{3}x + 4$
 - $y = x^2$
- Now cut out the left side (where $x < 0$) of the graph $y = -\frac{2}{3}x + 4$. Throw the other side away.
- Cut out the right side (where $x \geq 0$) of the graph $y = x^2$.
- Now take the two functions and tape them together at the y-axis.
This is the graph of

$$y = \begin{cases} -\frac{2}{3}x + 4 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

- Copy your graph here



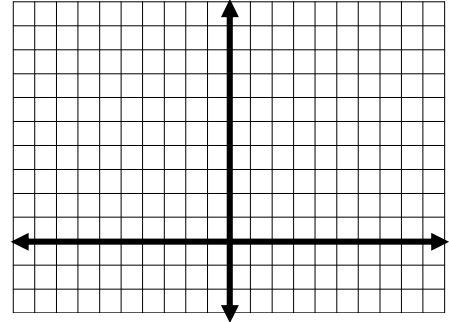
- The example above is called a **piecewise function** because the value of the function is made of two “pieces.”
- We also note that the function is **discontinuous** since it has one points ($x=0$) that has a “jump” discontinuities.
- Note: the function is **decreasing** on $(-\infty, 0)$ and **increasing** on $(0, \infty)$

Piecewise linear functions

A piecewise function definition like the one below gives a set of “rules” that define the output ($f(x)$) for any given input (x) value. Use the following function to answer questions 1 and 2.

$$F(x) = \begin{cases} x, & \text{if } x < 0 \\ x + 1, & \text{if } 0 \leq x < 5 \\ 2, & \text{if } x \geq 5 \end{cases}$$

- Find
 - $f(-2) =$
 - $f(0) =$
 - $f(1) =$
 - $f(3) =$
 - $f(5) =$
 - $f(6) =$
- Use these values to help you graph the function.



To plot piecewise functions, we can begin by graphing each of the functions (like $y = 0$, $y = x + 1$, and $y = 2$ above). Then “erase” part of the graph so that only part of the graph remains for the given domain piece. Endpoints are “closed” for included values and “open” for non-included values.

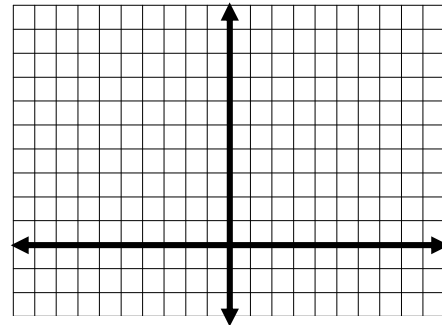
Mathematicians often choose to define more complicated functions using piecewise functions. This will greatly help us to model real-life situations

Try this

- Plot this piecewise function

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

- What non-piece function has the same graph as this?



Try These Graph the piecewise functions and determine if they are continuous

- $y = \begin{cases} x - 1, & \text{if } x \leq 0 \\ x^2, & \text{if } x > 0 \end{cases}$

- $y = \begin{cases} -2x + 1, & \text{if } x \leq 1 \\ -2x - 1, & \text{if } x > 1 \end{cases}$

- $y = \begin{cases} x + 1, & \text{if } x < 0 \\ 1, & \text{if } 0 \leq x < 1 \\ \sqrt{x}, & \text{if } x \geq 1 \end{cases}$

