Name: Date: Period:

1D: Average Rate of Change

Rates of Change

We have now learned how to describe the intervals in which functions increase or decrease, now we will consider *how much* they change in these intervals. We call this the ***Average Rate of Change***.

The average rate of change for a function is the ratio of the change in the y value and the change in the x value over a given interval. So, the average rate of change for a function $f(x)$ on the interval $[a,b]$ is given by:

$$Average\ Rate\ of\ Change = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

Gas Prices

Consider the table of average gas prices for one gallon of unleaded gas (U.S. EIA).

Price for 1 gal. Unleaded gas

Year	Avg. Price
1988	0.95
1990	1.16
1995	1.15
2000	1.51
2005	2.30
2008	3.27
2009	2.35
2010	2.79
2011	3.53

a) Find the average rate of change in the price of gas from 1990-2000.

b) Find the average rate of change in the price of gas from 2000-2011.

c) Compare these two averages and explain why you may think they are different.

Consider This.

Try it: Consider the function $f(x) = x^2$.

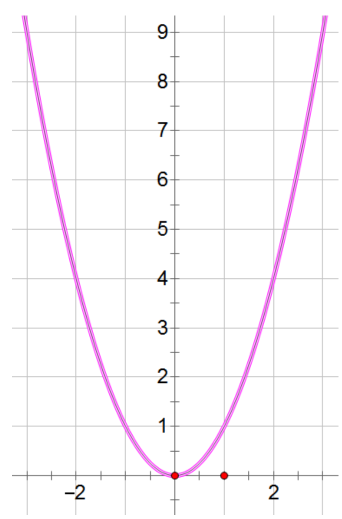
Find the rate of change for $g(x) = x^2$ over the following intervals

a. [0,1]

b. [1,2]

c. [2,3]

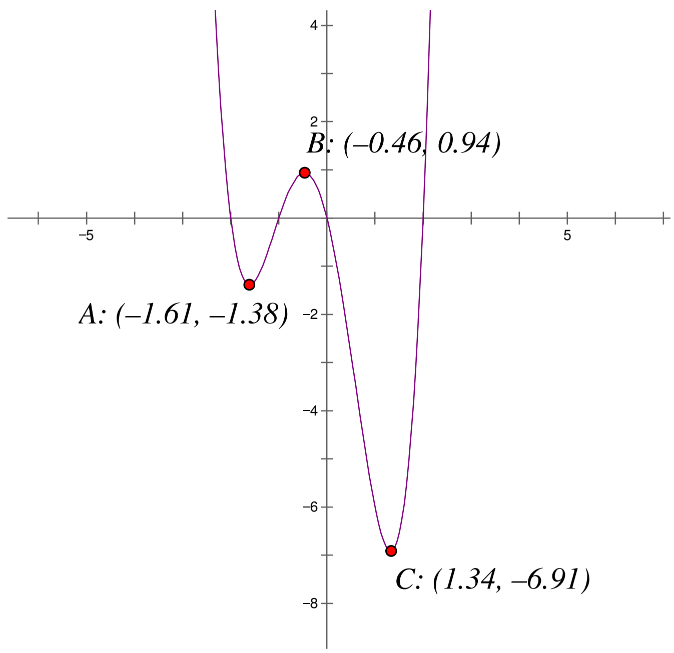
d. Draw secant lines on the graph to the right of $y = x^2$ to verify your answers to parts (a)-(d).



e. Use the formula to find the rate of change for $p(x) = x^2$ on $[1, 1+h]$. Simplify your answer.

<u>Key Correlation:</u> The average rate of change of $f(x)$ on $[a, b]$ is the ______________ of the secant line that passes through the points $(a, f(a))$ and $(b, f(b))$.

Intervals of Change



<u>Def.</u> **Increasing Interval**: the value of the function increases as the value of x increases.
If $x_1 < x_2$ then $f(x_1) < f(x_2)$.
That is, the rate of change is positive.

<u>Def.</u> **Decreasing Interval**: the value of the function decreases as the value of x increases.
If $x_1 < x_2$ then $f(x_1) > f(x_2)$
That is, the rate of change is negative.

<u>Def.</u> **Constant Interval**: the value of the function does not change as the value of x increases.
If $x_1 < x_2$ then $f(x_1) = f(x_2)$
That is, the rate of change is zero.

<u>Example:</u>

Find the intervals for which the function in the graph to the right is increasing or decreasing

Increasing on :

Decreasing on:

Example:

Graph the functions and find the intervals for which each function is increasing, decreasing, or constant:

a) $y = (x-3)^2$

b) $y = \frac{x}{1+x^2}$

Extrema:
Many functions reveal important information at the "peaks" and "valleys" that occur in their graphs where a function changes from increasing to decreasing or vice versa. These points are called extreme values or extrema.

- If the value of an extrema is less than its neighboring points, then it is a ***local minimum***
- If the value of an extrema is greater than its neighboring points, then it is a ***local maximum***
- If the value of an extrema is less than or greater than *all* the range values of the function, then it is
  called a ***global minimum*** or ***global maximum*** respectively.

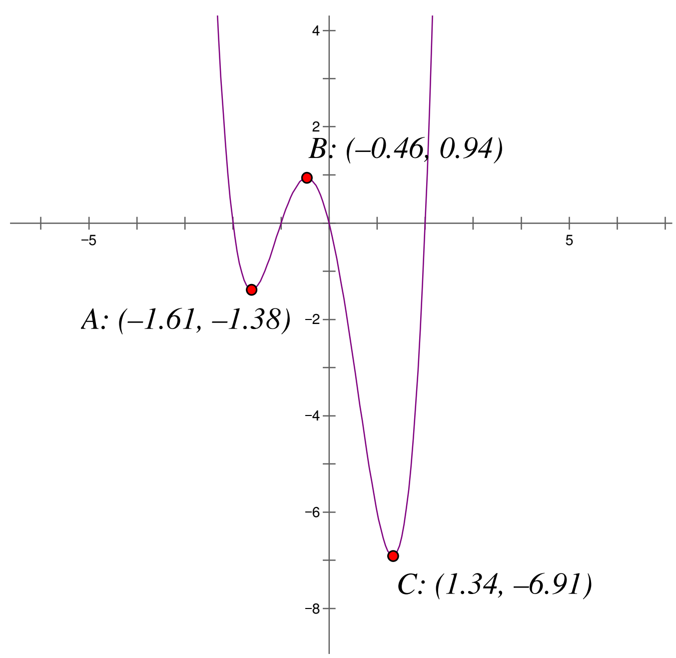
Example:

State whether each point in the graph to the right is a local maximum, local minimum, global maximum, or global minimum.

A:

B:

C:



Example:

Use a graphing utility to find the x value (to 2 decimal places) of all local maxima and minima and name the type. Then describe the increasing and decreasing intervals.

a) $y = x^3 - x^2 - 3x$

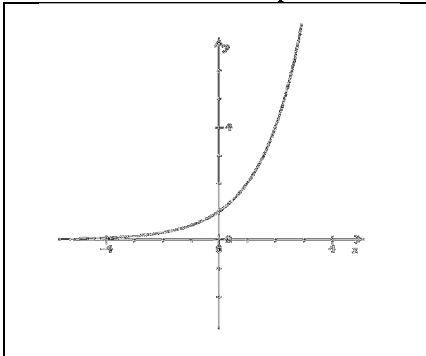
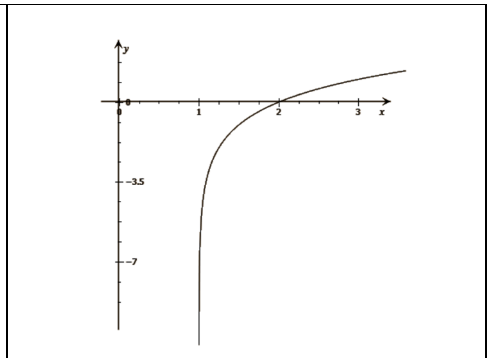
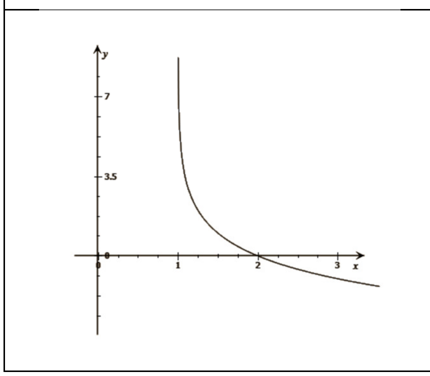
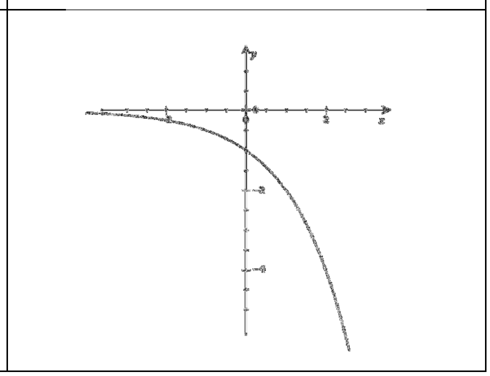
b) $y = x^4 + 3x^3 - x^2 - 4x$

Concavity

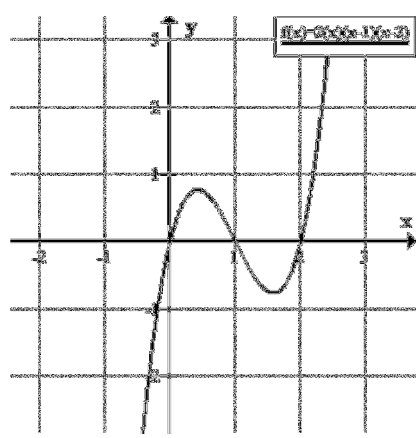
A final characteristic of that we will consider is the concavity of a graph. Now that we can find the Rate of Change, we like to know how the rate of change is changing.

- If the rate of change is *increasing* on an interval, then the graph is ***concave up.***
- If the rate of change is *decreasing* on an interval, then the graph is ***concave down***.
- A non-extrema point where a graph changes concavity is called an ***inflection point.***

Below are the possible types of concavity.

	Concave Up	Concave Down
Increasing		
Decreasing		

Try It: Use the graph to the right to approximate the intervals that belong in each box in the table



	Concave Up	Concave Down
Increasing		
Decreasing		