2B-2: Linear and Quadratic Modeling

Modeling is the process of representing a situation in algebraic terms using a combination of equations and graphs. Models can help us to better understand what is happening in system that we want to study.

Linear models:

The simplest model is one that can be represented with a linear function. There are three forms of linear functions that are commonly used in algebra applications:

- **Standard Form:** $ax + by = c$, when we know how x and y combine to make c.
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$, when given point (x_1, y_1) and slope m
- **Slope-Intercept Form:** $y = mx + b$, when we know the slope m and y-intercept b.

Make a Model.
Write a linear function to model each situation.

- Jane has a pay-per-minute phone. She pays \$.10 per minute per call.

- John has a pay per minute phone. He pays \$5.00 per month and \$.05 per minute.

- Rob's cellphone plan is a pre-pay plan that gives him 1000 minutes per month. He can send texts, but each text uses up 5 minutes of his talk-time (i.e. he can send 200 texts in a month and not talk at all if he wants, or any other combination in between). Write an equation that relates the minutes of calls, m, and the number of texts, t, that Rob can have in a month.

Quadratic Models:

Quadratic models are used for values that seem to follow the curved path of a quadratic parabola. These functions can be modeled using a quadratic of general form $f(x) = ax^2 + bx + c$.

Example. Consider the following set?

x	0	1	2	3	4	5
y	3	4	7	12	19	28

We observe the following differences:

Write a function that models these values.
(Hint: if it helps, make a row in the table for x^2, or graph the points).

Projectiles:

The height of a projectile (an object that is propelled into the air without any means of self-propulsion) is determined by the function

$$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$$

Where

$$s_0 = initial\ height$$
$$v_0 = initial\ vertical\ velocity,$$
$$g = acceleration\ of\ gravity \approx 32\frac{ft}{sec^2} \approx 9.8\frac{m}{sec^2}$$

The velocity of any projectile can be broken into two components: *vertical velocity* and *horizontal velocity*. For now, we will look at these separately. Note that when we model the height of a projectile with this function, we are *ignoring wind resistance*. Also, this *only measures vertical height* and disregards the horizontal position.

1. A bullet is shot in the air off of a 128 foot building with an initial vertical velocity of 320 ft. per second.

    a. Write a function to model the vertical height h as a function of time t.

    b. Use your graphing calculator to find the coordinates of the vertex (i.e. maximum) of the parabola.

    c. What is the amount of time at it will take the projectile to reach its maximum height.

    d. What is the maximum height of the projectile?

    e. Using the graph, find the time t that gives $s(t) = 0$.

    f. Suppose the horizontal velocity is 3000 ft. per second. Use the answer from part (e) to find the total horizontal distance the bullet will travel.