Pre-Calculus Name: SOLUTIONS Date: Period:

Assignment 1D: Rates of Change

Answer the following problems with as much detail, explanation, and work that is appropriate.

1. Use the formula to find the average rate of change for $f(x) = x^3$ on the intervals
   a. $[0,1]$

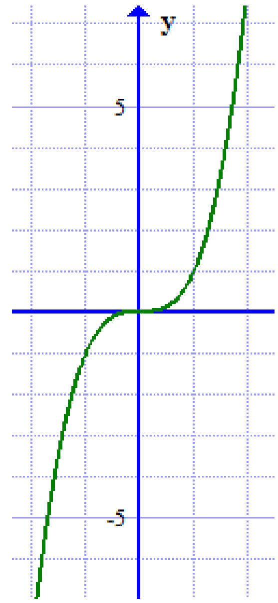
$$\frac{\Delta y}{\Delta x} = \frac{(1)^3 - (0)^3}{1 - 0} = 1$$

   b. $[-1,1]$

$$\frac{\Delta y}{\Delta x} = \frac{(-1)^3 - (1)^3}{-1 - 1} = 1$$

   c. $[-1,2]$

$$\frac{\Delta y}{\Delta x} = \frac{(2)^3 - (-1)^3}{(2) - (-1)} = 3$$



2. Show these rates of change for $f(x) = x^3$ graphically for each of the intervals above by drawing the secant lines on the graph to the right. Explain how these lines relate to the rates of change in #1

Find the average rate of change of each function on the interval specified.

3. $f(x) = x + 3$ on $[4,5]$

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(4)}{5 - 4} = \frac{\big((5) + 3\big) - \big((4) + 3\big)}{1} = \frac{8 - 7}{1} = 1$$

4. $g(x) = x^2 + 4$ on $[1,4]$

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{((4)^2 + 4) - ((1)^2 + 4)}{3} = \frac{20 - 5}{3} = \frac{15}{3} = 5$$

5. $h(x) = x^2 + 2x$ on $[-5,-3]$

$$\frac{\Delta y}{\Delta x} = \frac{f(-5) - f(-3)}{(-5) - (-3)} = \frac{\big((-5)^2 + 2(-5)\big) - \big((-3)^2 + 2(-3)\big)}{-2} = \frac{15 - 3}{-2} = -6$$

6. $p(t) = \frac{x^3-2x}{x^2+1}$ on $[-2,1]$

The inputs $t = -2$ and $t = 1$ when put into the function $p(t)$ produce the points $\left(-2,-\frac{4}{5}\right)$ and $(1,-\frac{1}{2})$. The average rate of change between these two points is

$$\frac{\Delta y}{\Delta x} = \frac{p(1)-p(-2)}{(1)-(-2)} = \frac{-\frac{1}{2}-\left(-\frac{4}{5}\right)}{3} = \frac{-\frac{5}{10}+\frac{8}{10}}{3} = \frac{\frac{3}{10}}{3} = \frac{3}{10} * \frac{1}{3} = \frac{1}{10}$$

Find the average rate of change of each function on the interval specified. Your answers will be expressions involving a parameter (b or h).

7. $f(x) = x^3 - 3x$ on $[4, b]$

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(4)}{b-4} = \frac{(b^3 - 3b) - \left((4)^3 - 3(4)\right)}{b-4} = \frac{b^3 - 3b - 52}{b-4}$$

8. $g(x) = 3x^2 - 2$ on $[x, x+h]$

$$\frac{\Delta y}{\Delta x} = \frac{(3(x+h)^2-2)-(3x^2-2)}{(x+h)-x} = \frac{(3(x+h)^2-2)-(3x^2-2)}{h} = \frac{3(x+h)^2-2-3x^2+2}{h} = \frac{3(x+h)^2-3x^2}{h} =$$

$$\frac{3(x^2+2hx+h^2)-3x^2}{h} = \frac{3x^2+6hx+3h^2-3x^2}{h} = \frac{6hx+3h^2}{h} = 6x + 3h = 3(2x+h).$$

9. Graph $h(x) = x^5 + 5x^4 + 10x^3 + 10x^2 - 1$ on your calculator.

   a. Find all the local extrema of the function and state what type it is.

   The function has a local minimum at $(0, -1)$, and a local maximum at $(-2, 7)$.

   b. Find the increasing intervals.

   From the graph, we can see that the function is decreasing on the interval $(-2, 0)$, and increasing on the intervals $(-\infty, -2)$ and $(0, \infty)$.

   c. Find the decreasing intervals.

   the function is decreasing on the interval $(-2, 0)$

   d. ***Challenge:*** Define all the intervals that are concave up and concave down. Approximate inflection points.

We can estimate that the function is concave down on the interval $(-\infty, -1)$, and concave up on the intervals $(-\infty, -1)$ and $(0, \infty)$. This means there is an inflection point at $x = -1$.