Pre-Calculus

Name: SOLUTIONS Date: Period:

Assignment 2C

For each polynomial function below, write in standard form, state the degree, find the y-intercept, find the number of *possible* zeros and turning points (a.k.a. extrema), and describe the end behavior *without graphing*. Then verify the end behavior with your graphing calculator and find the *actual* number of zeros and turning points of the function. *Sketch* a rough picture of your graph.
Write your answers as a complete thought (the first problem is modeled for you).

1. $y = 2x^3 - 2 + 3x^4 - 3x^2$

$$y = 3x^4 + 2x^3 - 3x^2 - 2$$

This polynomial has degree 4 with a

y-intercept of -2 . There are 4

possible zeros and3 possible turning points.

As $x \to -\infty, y \to \infty$.
As $x \to \infty, y \to \infty$.

This function actually has 2 zeros

and 3 turning points.

2. $y = -x^5 - 3x^6 - 4x^4 + 3x^5 + 10$

$y = -3x^6 + 2x^5 - 4x^4 + 10$

This polynomial has degree 6 with a

y-intercept of 10 . There are 6

possible zeros and 5 possible turning points.

As $x \to -\infty, y \to -\infty$.
As $x \to \infty, y \to -\infty$.

This function actually has 2 zeros

and 1 turning points.

3. $y = 2 + x^4 - 10\,x^2 - 5\,x - 3\,x^3$

$y = x^4 - 3x^3 - 10x^2 - 5x + 2$

This polynomial has degree 4 with a

y-intercept of 2 . There are 4

possible zeros and 3 possible turning points.

As $x \to -\infty, y \to \infty$.
As $x \to \infty, y \to \infty$.

This function actually has 2 zeros

and 3 turning points.

4. $y = 1 + 4x^3 - x^4 - 6x$

$y = -x^4 + 4x^3 - 6x + 1$

This polynomial has degree 4 with a

y-intercept of 1 . There are 4

possible zeros and 3 possible turning points.

As $x \to -\infty, y \to -\infty$.
As $x \to \infty, y \to -\infty$.

This function actually has 4 zeros

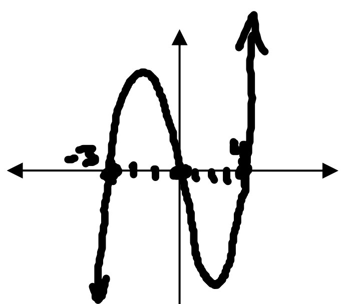
and 3 turning points.

Use factoring to find the zeros of the functions (list their coordinates), state the multiplicity of each zero, then *sketch* a graph of each function with approximate scale. Check with your calculator.

5. $f(x) = x^3 - x^2 - 12\,x$

$f(x) = x(x^2 - x - 12)$
$= x(x-4)(x+3)$

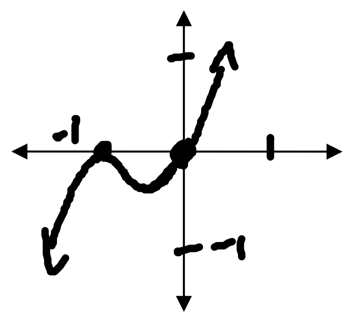
Zeros: $x = 0, 4, -3$
All w/ mult. 1



6. $g(x) = 3\,x^3 + 6\,x^2 + 3\,x$

$g(x) = 3x(x^2 + 2x + 1)$
$= 3x(x+1)^2$

Zeros: $x = 0$ Mult. 1
$x = -1$ Mult. 2



7. $h(x) = x^4 - x^2 - 12$

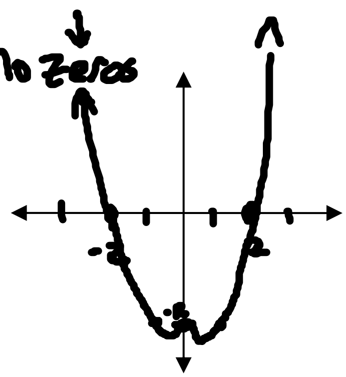
Let $u = x^2$

$h(x) = u^2 - u - 12$
$= (u-4)(u+3)$

$u = 4$ or $u = -3$
$x^2 = 4$ $\qquad$ $x^2 = -3$

Zeros $\boxed{x = \pm 2 \text{ (Mult. 1)}}$

no zeros



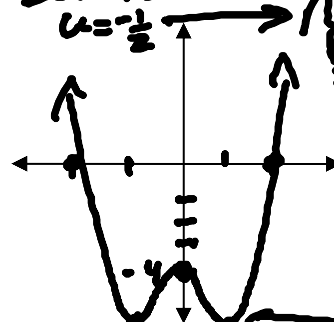
8. $j(x) = 2\,x^4 - 7\,x^2 - 4$

$u = x^2$

$j(x) = 2u^2 - 7u - 4$
$ac = -8 \rightarrow = (2u^2 - 8u) + (1u - 4)$
$= 2u(u-4) + 1(u-4)$
$= (u-4)(2u+1)$

$u = 4$ or $2u + 1 = 0$
$x^2 = 4$ $\qquad$ $u = -\frac{1}{2} \rightarrow$ No Real zeros

$\boxed{x = \pm 2 \text{ (Mult 1)}}$



How do I know this?

Use the problems above to solve these inequalities (note: the zeros above are the boundary points)

9. $x^3 - x^2 - 12\,x \geq 0$

By graph, zeros at $0, -3, 4$

$[-3, 0] \cup [4, \infty)$

10. $x^4 - x^2 - 12 < 0$

$(-2, 2)$