2C: Graphing Polynomial Functions

We have worked with polynomials to determine their general form and some characteristics of their graphs. In this lesson, we will begin to analyze the shape of a polynomials graphs by finding intercepts and considering factors to get a good picture of the function. First let's review a few polynomial facts.

Polynomials and End behavior

Definition – a **Polynomial** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0, \qquad a_n \neq 0$$

- Each monomial is called a ***term***
- The largest power n is called the ***degree***
- A polynomial with powers written in descending order is called ***standard form***
- The numbers $a_n, a_{n-1}, \ldots, a_0$ are called the ***coefficients*** of the polynomial
- The term $a_n x^n$ is called the ***leading term***, a_n is the ***leading coefficient***, and a_0 is the ***constant term***.

Theorem: ***Polynomial Extrema and Zeros***

A polynomial function of degree n has at most $n-1$ local extrema and at most n zeros.

End (Long run) behavior of polynomials

We can know what the graph of a polynomial can look like just by knowing its degree and its leading coefficient as shown below.

Degree	***Leading Coefficient***	***As*** $x \to -\infty$	***As*** $x \to \infty$	***Picture***
Even	Positive	$y \to \infty$	$y \to \infty$	
Even	Negative	$y \to -\infty$	$y \to -\infty$	
Odd	Positive	$y \to -\infty$	$y \to \infty$	
Odd	Negative	$y \to \infty$	$y \to -\infty$	

Finding Zeros

Definition: The number k is a **zero** of a function f if and only if $f(k) = 0$.

Method: To find the zeros of a function, we set $f(x) = 0$ and solve for x by factoring. To factor we first look for common monomials, then factor out linear divisors $(x - k)$ using synthetic division, then use other methods such as grouping or quadratic methods.

Example: Find the zeros of $f(x) = (x - 4)(x + 3)(2x - 6)$.

When a function is factored as it is in the previous example, it's easy to find the zeros. However, the same function can be written as $f(x) = 2\,x^3 - 8\,x^2 - 18\,x + 72$, and it's more difficult to see the zeros. For polynomials in standard form, we can use our calculators to find the zeros.

Example: Use your calculator to find the zeros of $f(x) = x^3 - 6\,x^2 + 6$ to two decimal places.

Sometimes polynomials can be factored easily like the following example.

Example: Use factoring to find all the zeros of $f(x) = 2\,x^3 + 12\,x^2 + 18\,x$

Example: Use factoring to find all the zeros of $k(x) = x^4 - 7\,x^2 - 18$
This equation is in "quadratic form"

Repeated zeros: If the factorization of a polynomial function includes a factor of $(x - k)^m$, then the zero k is said to have "multiplicity m".

- If a real zero c has odd multiplicity, the graph crosses the x axis at $(c, 0)$
- If a real zero c has even multiplicity, the graph does not cross the x axis at $(c, 0)$

Example: State the multiplicity of the zeros of $f(x) = 2\,x^3 + 12\,x^2 + 18\,x$.

Example: $f(x)$ is a polynomial of degree 4. It has a root of multiplicity 2 at x=1, and roots of multiplicity 1 at x=0 and x=-2. It also goes through the point (5,448). Find the formula for $f(x)$.

Graphing and Transforming Basic Polynomials

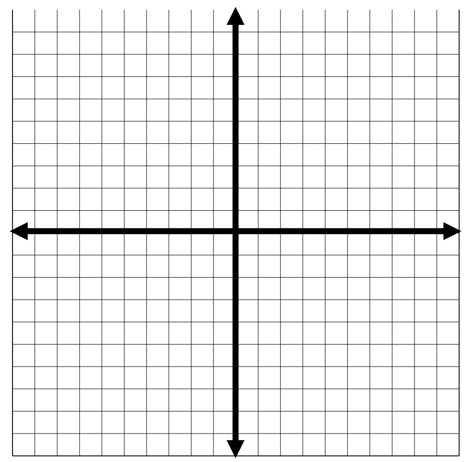
Transforming Polynomials

The following transformations will affect the graph of a polynomial $f(x)$:

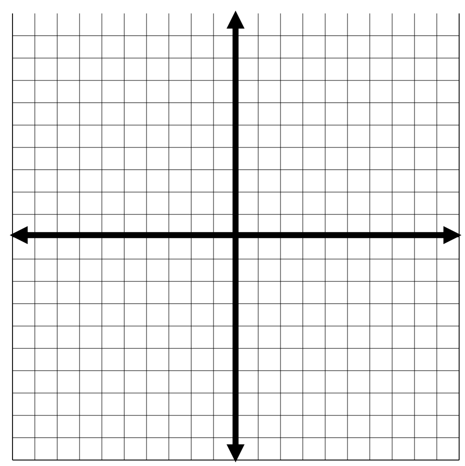
- $g(x) = f(x - h)$ will translate the graph of $f(x)$ horizontally h units.
- $g(x) = f(x) + k$ will translate the graph of $f(x)$ vertically k units.
- $g(x) = c \cdot f(x)$ will stretch the graph of $f(x)$ by a factor of c.
  If $c < 0$, the graph will be reflected across the y-axis.

Explore. Graph the following functions by hand and check with your graphing calculator, then simplify the right side to write it in standard form.

a) $g(x) = \frac{1}{3}(x - 1)^3 - 3$.
Hint: Use Transformations of $f(x) = x^3$ to graph



b) $h(x) = (x - 2)(x + 1)(x + 4)$.
Hint: Find zeros, then consider the degree of the polynomial.



<u>**Consider This**</u> Use the graphs above to answer these inequalities.
To do this, list the intervals of x-values that make the inequality true.

a) $\frac{1}{3}(x - 1)^3 - 3 > 0$

b) $(x - 2)(x + 1)(x + 4) \leq 0$