Assignment 2D-SOLUTIONS

After completing lesson 3C, complete the assignment below.

Simplify the following expressions.

1. $(3i)^2$

   $\mathbf{-9}$

2. $(2+3i)(1-4i)$

   $\mathbf{14-5i}$

3. $(x+i)(x-i)$

   $\mathbf{x^2+1}$

4. $(x-\sqrt{3}i)(x+\sqrt{3}i)$

   $\mathbf{x^2+3}$

5. $(\boldsymbol{a}+\boldsymbol{b}i)(\boldsymbol{a}-\boldsymbol{b}i)$

   $\mathbf{a^2+b^2}$

6. Complex conjugates are two complex numbers of the form $\boldsymbol{a}+\boldsymbol{b}i$ and $\boldsymbol{a}-\boldsymbol{b}i$.
   What happens everytime you multiply two complex conjugates (as in problems 3-5)?

   The imaginary unit is canceled out.

7. Consider a polynomial $f(x)=x^4+x^3-4x^2+2x-12$ that can be written with a linear factorization of the form $f(x)=(x-a)(x-b)(x-c)(x-d)$.
   If two of these factor contain complex numbers, explain why do we know that two of the terms must be complex conjugates? (Hint: Consider your answer to #6)

   Since there are no imaginary units, i, in the function, the imaginary numbers must be conjugates to cancel each other out when multiplied.

8. The polynomial $f(x)=x^4+x^3-4x^2+2x-12$ has zeros at $x=-3$ and $x=2$.
   a. Find a real factorization of $f(x)$.

      $$\boldsymbol{f(x)=(x-2)\,(x+3)\,\left(x^2+2\right)}$$

   b. Find the complex zeros of $f(x)$.

      $\boldsymbol{x=2,-3,i\sqrt{2},-i\sqrt{2}}$

   c. Write the linear factorization (with real and complex factors) of $f(x)$.

      $$\mathbf{f(x)=(x-2)(x+3)\left(x-i\sqrt{2}\right)\left(x+i\sqrt{2}\right)}$$

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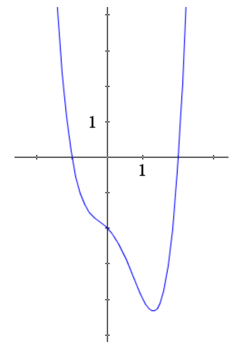
9. State the ***Fundamental Theorem of Algebra***.

Every nth degree polynomial has exactly n complex zeros

10. Consider the quartic (4th degree) function graphed to the right.



a. How many real zeros does the function have? Explain.

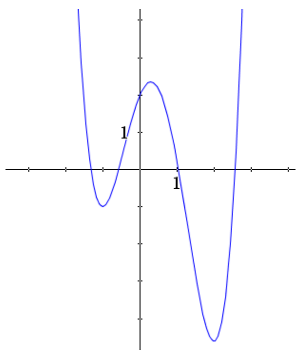
$\boldsymbol{2 \text{ real zeros because it has two } x - \text{intercepts}}$

b. Does the function have any imaginary zeros? If so, how many? Explain how you know.

$\boldsymbol{2 \text{ imaginary zeros because it must have 4 complex zeros.}}$

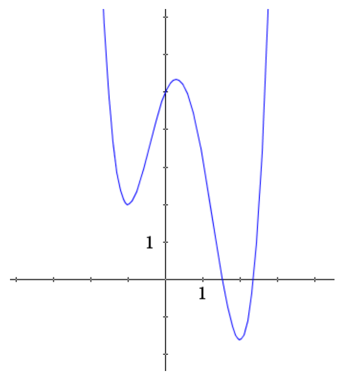
11. For each quartic function below, state how many real zeros (including repeated zeros) and how many imaginary zeros it has.

a)



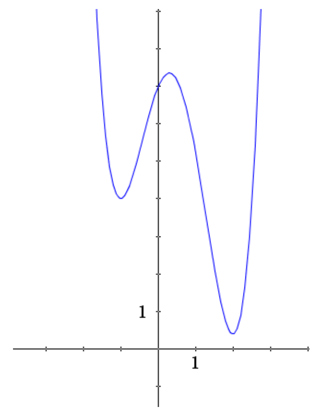
$\boldsymbol{4 \text{ real}, 0 \text{ imag.}}$

b)



$\boldsymbol{2 \text{ real}, 2 \text{ imag.}}$

c)



$\boldsymbol{0 \text{ real}, 4 \text{ imag.}}$

Completely factor the polynomials. You may use your calculator (to start), synthetic division, factoring, or the quadratic formula. Leave answers as exact answers in simplified form.

12. $x^4 + 2x^3 + x^2 + 8x - 12$

$\boldsymbol{(x-1)(x+3)(x^2+4)}$

$\boldsymbol{(x-1)(x+3)(x+2i)(x-2i)}$

13. $x^4 - 2x^3 - 6x^2 - 7x - 4$

$\boldsymbol{(x+1)(x^3 - 3x^2 - 3x - 4)}$

$\boldsymbol{(x+1)(x-4)(x^2 + x + 1)}$

$$\boldsymbol{(x+1)(x-4)\left(x - \frac{-1 + i\sqrt{3}}{2}\right)\left(x - \frac{-1 - i\sqrt{3}}{2}\right)}$$

Solve the equations. You may use your calculator (to start), synthetic division, factoring, or the quadratic formula. Leave answers as exact answers in simplified form.

14. $2\,x^4 + 7\,x^3 - 4\,x^2 - x - 4 = 0$

$$(x-1)\,(x+4)\left(2x^2+x+1\right)=0$$

$$x = \frac{-1 \pm i\sqrt{7}}{4}, 1, -4$$

15. $4\,x^5 - 16\,x^4 + 7\,x^3 = -12\,x^2 - 3\,x - 18$

$$4x^5 - 16x^4 + 7x^3 + 12x^2 + 3x + 18 = 0$$

$$x = -1 \text{ or } x = 2 \text{ or } x = 3, x = \frac{\sqrt{3}}{2}i, x = \frac{-\sqrt{3}}{2}i$$