

2A: Combining and Modeling with Transformations

In this lesson, we will be investigating the effects of several different types of transformations of a function. We have four different types of transformations:

- **rotation** about a point
- **reflection** across a line
- vertical or horizontal slide **translation**
- **dilation** by a certain scale factor

When graphing quadratic equations it is useful to know what kinds of transformations can be made by altering the quadratic equation.

Translations

Let's begin by figuring out how to translate the graph of a function.

Exploration 1

Consider one of your basic "toolkit" functions:

$$f(x) = c, f(x) = x, f(x) = x^2, f(x) = x^3, f(x) = |x|, f(x) = \frac{1}{x}, f(x) = 1/x^2$$

1. Graph one of these functions on your graphing utility.
2. Now change the function by adding a constant d to the function to see what happens to the graph of $f(x) + d$. Change the value of d to find a pattern in the change.
3. Repeat this for several other functions.
How does d affect the shape or position of the graph?

4. Now change the function by adding a constant d to the function to see what happens to the graph of $f(x - c)$. Change the value of c to find a pattern in the change.
5. Repeat this for several other functions.
How does c affect the shape or position of the graph?

Example 1:

Tables of values for $f(x)$, $g(x)$, and $h(x)$ are given below. Write $g(x)$ and $h(x)$ as transformations of $f(x)$.

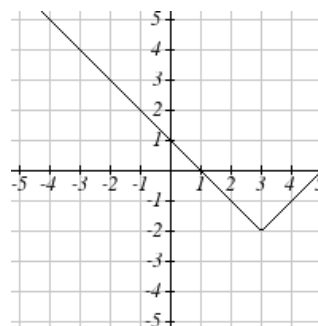
x	-2	-1	0	1	2
f(x)	-2	-1	-3	1	2

x	-1	0	1	2	3
g(x)	-2	-1	-3	1	2

x	-2	-1	0	1	2
h(x)	-1	0	-2	2	3

Example 2:

- a) Write an equation of the function in the graph using translations of a basic function.
- b) Write an equation for the graph of $f(x) = x^4$ if it is translated and up 6 units.



left 5 units

Stretching and Compressing

Let's begin by figuring out how to stretch a graph

Exploration 1

Consider again one of your basic "toolkit" functions:

$$f(x) = c, f(x) = x, f(x) = x^2, f(x) = x^3, f(x) = |x|, f(x) = \frac{1}{x}, f(x) = 1/x^2$$

1. Graph one of these functions on your graphing utility.
2. Now change the function by adding a constant d to the function to see what happens to the graph of $a \cdot f(x)$. Change the value of a to find a pattern in the change.
3. Repeat this for several other functions.
How does a affect the shape or position of the graph if $a > 1$?

How does a affect the shape or position of the graph if $a < 1$?

How does the graph change if a is negative?

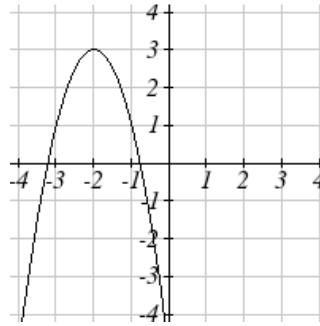
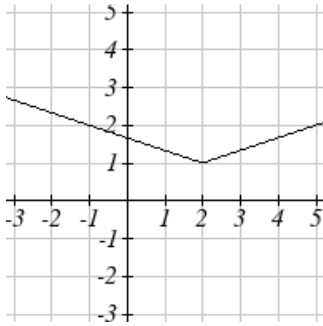
4. Now change the function by adding a constant d to the function to see what happens to the graph of $f(b \cdot x)$. Change the value of b to find a pattern in the change.
5. Repeat this for several other functions.
How does b affect the shape or position of the graph if $b > 1$?

How does b affect the shape or position of the graph if $b < 1$?

How does the graph change if b is negative?

Example 3.

Write an equation for each of the functions that is graphed below:



Translating and Stretching functions:

Given a function $f(x)$, the parameters a , b , c , and d in the function $g(x) = a \cdot f(b \cdot x + c) + d$ will change the graph of $f(x)$ as follows:

a:

b:

c:

d: