# 3A: Rational Expressions and Equations

A ***rational expression or equation*** is a one that has a variable in the denominator of a fraction. You are familiar with techniques for working with polynomials and solving polynomial equations, so all we need to do to solve a rational equation is to change it into a polynomial equation.

## Simplifying Rational Expressions

When working with rational expression, we just need to remember all the rules for operations with fractions. Multiplying fractions and rational expressions is easy since we can just multiply numerators and denominators, but addition of rational expressions (and subtraction) gets a little more difficult as we need to find a common denominator.

**Try These:** Simplify these using by multiplying and then simplifying. Using cross-cancelation techniques will be very helpful!

**Cross Canceling:** How does this demonstrate that cross-cancelling always works when multiplying fractions?

$$\frac{15}{28}\cdot\frac{21}{20}\cdot\frac{8}{18}=\frac{3\cdot 5}{4\cdot 7}\cdot\frac{3\cdot 7}{4\cdot 5}\cdot\frac{2\cdot 4}{3\cdot 3\cdot 2}=\frac{3\cdot 5\cdot 3\cdot 7\cdot 2\cdot 4}{4\cdot 7\cdot 4\cdot 5\cdot 3\cdot 3\cdot 2}=\frac{2\cdot 3\cdot 3\cdot 4\cdot 5\cdot 7}{2\cdot 3\cdot 3\cdot 4\cdot 5\cdot 7\cdot 4}=\frac{2\cdot 3\cdot 3\cdot 4\cdot 5\cdot 7}{2\cdot 3\cdot 3\cdot 4\cdot 5\cdot 7}\cdot\frac{1}{4}=\frac{1}{4}$$

***Simplify***

a) $\frac{200}{27}\cdot\frac{12}{160}\cdot\frac{9}{20}$

b) $\frac{6x}{25}\cdot\frac{5}{8x^2}\cdot\frac{20x^4}{12}$

c) $\frac{x+3}{x-5}\cdot\frac{x^2-4x-5}{x+1}$

**Adding Rational Expressions**

Remember, when we add fractions, we need to find a common denominator.

***Simplify the expression by finding a common denominator.***

a) $\frac{2}{x}+\frac{x}{5}-\frac{3}{x+1}$

b) $\frac{1}{x+3}-\frac{1}{x-3}+\frac{3}{x^2-9}$

## Solving Rational Equations

The key to solving rational equations is a few simple steps:

1. Find the ***least common denominator*** for all terms, then
2. ***Multiply both sides*** of the equation by the common denominator, then
3. ***Solve*** the remaining equation using standard techniques, then
4. ***Check for extraneous solutions*** by substituting answers back into original equation. *Specifically, make sure that there are no conflicts with the domain restrictions.*

***Example.***

***Solve.***

$\frac{1}{x+3}+\frac{2}{x}=\frac{3}{x^2+3x}$

$$\frac{x}{x+3}+\frac{2}{x}=\frac{6}{x(x+3)}$$

➢ *Factor all denominators.*

$$x(x+3)\left(\frac{x}{x+3}+\frac{2}{x}\right)=\left(\frac{6}{x(x+3)}\right)\cdot x(x+3)$$

➢ *Find the common denominator by multiply all unique denominator factors.* $x(x+3)$

$$\frac{x\cdot x(x+3)}{x+3}+\frac{2\cdot x(x+3)}{x}=\frac{6\cdot x(x+3)}{x(x+3)}$$

➢ *Make sure you distribute to each term.*

$$x\cdot x+2(x+3)=6$$
$$x^2+2x+6=6$$
$$x^2+2x=0$$
$$x(x+2)=0$$
$$x=0 \;\; or \;\; x=-2$$

➢ *Cancel factors that you can, then solve.*

*Since* $x\neq 0$ *for the second term on left-hand side* $\frac{2}{x}$*, we must eliminate this solution.*

***Solution set:*** $\mathbf{x=-2}$

➢ *Check solutions with denominators to avoid domain restrictions.*

***Try it.*** *Solve these rational equations. Find all complex solutions.*

a) $\frac{3}{x^2+3x+2}-\frac{x}{x+2}=\frac{2}{x+1}$

b) $\frac{x}{2}+\frac{1}{2x-4}=\frac{1}{x^2-2x}$