Pre-Calculus Name: Date: Period:

3C: Advanced Rational Functions

Rational functions can behave in many different ways. Sometimes they look very familiar, and sometimes they act in very unique, unexpected ways.

Not as complicated as it looks!

Some rational functions simplify to make non-rational functions, but we still need to be careful to watch out for domain restrictions.

Two Vertical Asymptotes

When there are more than one factor in the denominator that cannot be simplified, each factor creates a vertical asymptote. In between these asymptotes, the graph will be similar to some polynomials that we have worked with.

Remember, the horizontal asymptote only affects the end-behavior of the graph. The graph in the middle may cross this asymptote.

Two asymptotes: Graph this function after considering the key features.

$$g(x) = \frac{1}{x^2 - 4}$$

a) Find the domain restrictions.
   State the domain of the function.

b) Are there any holes in this graph? Explain.

c) Find the equation(s) of the vertical asymptote(s).Find these limits:

$$\lim_{x \to -2^-} \frac{1}{x^2 - 4} = \qquad \lim_{x \to 2^-} \frac{1}{x^2 - 4} =$$

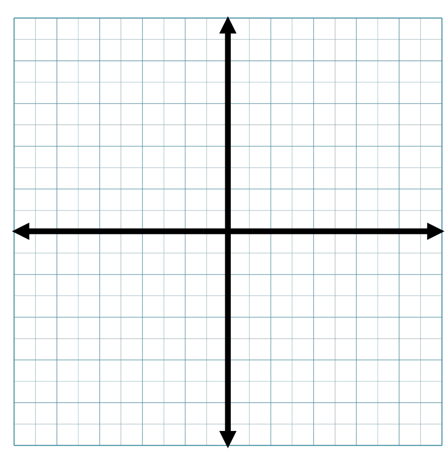
$$\lim_{x \to -2^+} \frac{1}{x^2 - 4} = \qquad \lim_{x \to 2^+} \frac{1}{x^2 - 4} =$$

d) Divide the numerator and denominator by x^2 and write the simplified result.

e) Find the limit of this function in (d) as $x \to \infty$
   Use this to find the horizontal asymptote.

f) Find the coordinates of all the intercepts.

g) Graph the function $g(x) = \frac{1}{x^2 - 4}$

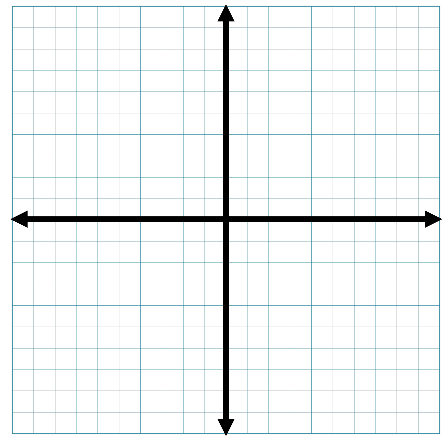


Two Asymptotes, odd numerator degree:

Graph this function using the steps from above. $h(x) = \frac{x}{x^2-x-6} = \frac{x}{(x-3)(x+2)}$

Vertical Asymptotes:

Horizontal Asymptote:



$\lim_{x\to -2^-} h(x) =$ $\qquad$ $\lim_{x\to 3^-} h(x) =$

$\lim_{x\to -2^+} h(x) =$ $\qquad$ $\lim_{x\to 3^+} h(x) =$

Slant Asymptotes

When the degree of the numerator is greater than the degree of the denominator in the *simplified form* of a rational function, the function will have a *slant asymptote* instead of a vertical asymptote.

To find the slant asymptote, you can use either method introduced in the previous lesson:

Method 1: Use long-hand division to divide the fraction. The quotient is the slant asymptote.
Method 2: Divide the numerator and denominator by the highest power of x in the denominator. As $x \to \infty$, the fractions with x in the denominator become zero. Ignore these fractions and you have the slant asymptote equation.

Try it out Graph this rational function by finding the asymptotes, the intercepts, and testing any other points needed.

$$j(x) = \frac{2x^2+1}{x+1}$$

a. State the domain restrictions for this function.
State the domain of the function.

b. What is the equation of the vertical asymptote?

c. Now let's consider some limits as x.

$$\lim_{x\to -1^-} j(x) = \qquad\qquad \lim_{x\to -1^+} j(x) =$$

$$j(x) = \frac{2x^2 + 1}{x + 1}$$

d. Let's find the slant asymptote. Divide the numerator and denominator by x (the largest power of x in the denominator.)

e. Now consider the limit of y as $x \to \infty$. The fractions disappear and the behavior of the function is the leftover equation. This is your slant asymptote.

f. Now find the coordinates of the x-intercept and the y-intercept (if they exist).

g. Now graph the function. Draw your asymptotes and your intercepts.