4B: Graphing Logarithmic Functions

Logarithmic Functions – The Inverse of an Exponential Function

The inverse of an exponential function $f(x) = b^x$ is called the **logarithmic function with base *b***, which we write as $\log_b(x)$ or $\log_b x$. Written as inverses, we say if $b > 0$ and $b \neq 1$

$$\text{if } f(x) = b^x, \text{then } f^{-1}(x) = \log_b x$$

Remember: <u>Converting between Logarithmic and Exponential form:</u>

If $x > 0$ and $0 < b \neq 1$, then $y = \log_b(x)$ is equivalent to $b^y = x$.

That is… ***<u>The log is the power</u>***!

<u>Try these:</u> Write each logarithmic equation in exponential form to solve for the variable.

a) $\log_2(8) = y$

b) $\log_2\left(\frac{1}{8}\right) = y$

c) $\log_5(25) = y$

d) $\log_3\left(\frac{1}{9}\right) = y$

<u>Consider This:</u> Can you find the logarithm of a non-positive number? That is, could you find the following logarithms: $\log_2(0) = ?$, $\log_2(-8) = ?$ *Explain why or why not.*

Graphing Logarithmic Functions

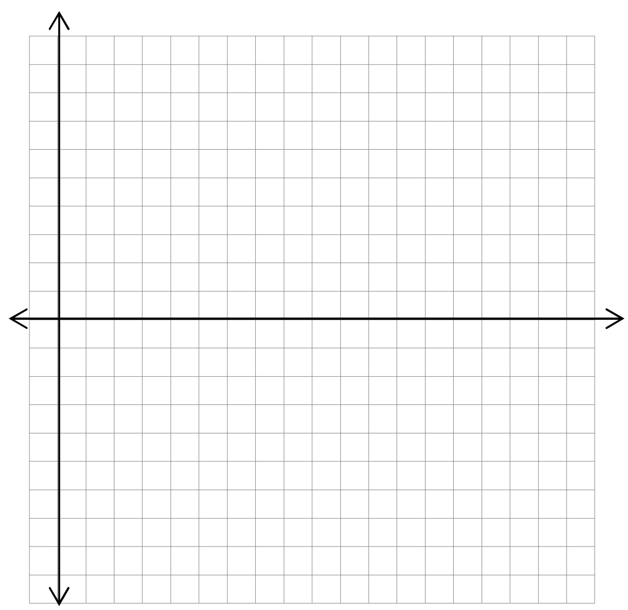
Complete the tables and use them graph the functions by hand on the same graph.

$y = \log_2 x$

x	y
16	
8	
4	
2	
1	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{8}$	

$y = \log_3 x$

x	y
9	
3	
1	
$\frac{1}{3}$	
$\frac{1}{9}$	



Translating Common and Natural Logarithms

Link to Desmos Logarithm Exploration:

Exploring Transformations

Now let's explore some transformations of exponential graphs. Use dynamic Algebra software (like Desmos), graph the following using sliders for the variable. Describe the change in the shape of the graph on the given interval.

- Graph the function $y = a\log_2 x$ using a slider for a on $[-10,10]$
- Graph the function $y = \log_2(bx)$ using a slider for b on $[-10, 10]$
- Graph the function $y = \log_2(x + c)$ using a slider for c on $[-10,10]$
- Graph the function $y = \log_2(x) + d$ using a slider fro d on $[-10,10]$

Transformations of Logarithmic Functions:

The values of a, b, c and d affect the graph of $\boldsymbol{y = a\log\big(b(x + c)\big) + d}$ as follows:

a: Vertical stretch/shrink. If a is negative, it is reflected across the x-axis

b: Horizontal stretch/shrink. If b is negative, the graph is reflected across the y-axis.

c: Horizontal shift.

if $c>0$, graph moves left; if $c<0$, graph moves right.

d: Vertical shift.

if $d>0$, graph moves up; If $d<0$, graph moves down.

Finding the Domain of a Logarithmic Function

We discovered above that we cannot find the logarithm of a non-positive number (0 or negatives). We can use this idea to determine what the domain is of a logarithmic function by finding the set of numbers that forces the value inside the logarithm to be positive.

<u>Try it:</u> State the domain of each function.

a) $y = \log_3(x)$
b) $y = \log_5(x + 4)$
c) $y = \log(4x)$
d) $y = \ln(x - 5) + 10$
e) $y = 7\ln(2x + 5)$