4B: Graphing Logarithmic Functions

Logarithmic Functions – The Inverse of an Exponential Function

The inverse of an exponential function $f(x) = b^x$ is called the **logarithmic function with base *b***, which we write as $\log_b(x)$ or $\log_b x$. Written as inverses, we say if $b > 0$ and $b \neq 1$

if $f(x) = b^x$, then $f^{-1}(x) = \log_b x$

Remember: Converting between Logarithmic and Exponential form:

If $x > 0$ and $0 < b \neq 1$, then $y = \log_b(x)$ is equivalent to $b^y = x$.

That is… ***The log is the power***!

Try these: Write each logarithmic equation in exponential form to solve for the variable.

a) $\log_2(8) = y$

b) $\log_2\left(\frac{1}{8}\right) = y$

c) $\log_5(25) = y$

d) $\log_3\left(\frac{1}{9}\right) = y$

Consider This: Can you find the logarithm of a non-positive number? That is, could you find the following logarithms: $\log_2(0) = ?$, $\log_2(-8) = ?$ *Explain why or why not.*

Graphing Logarithmic Functions

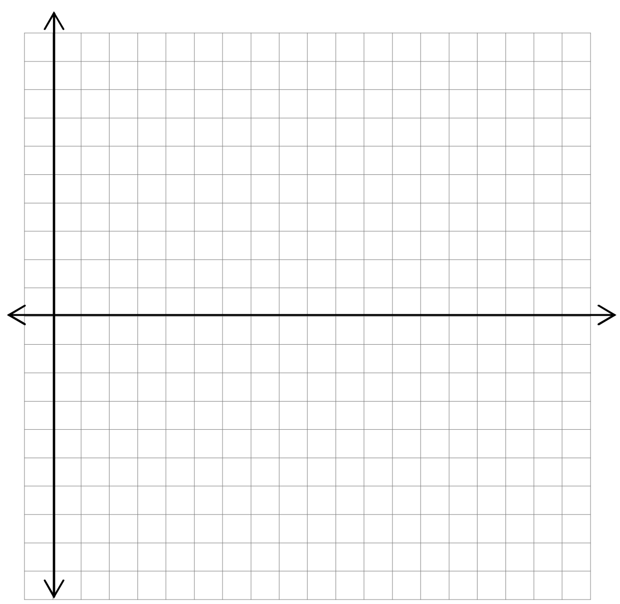
Complete the tables and use them graph the functions by hand on the same graph.

$y = \log_2 x$

x	y
16	
8	
4	
2	
1	
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{8}$	

$y = \log_3 x$

x	y
9	
3	
1	
$\frac{1}{3}$	
$\frac{1}{9}$	



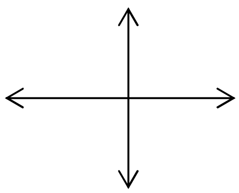
Translating Common and Natural Logarithms

We often use logarithms with base 10. We write these logarithms as $y = \log x$ without writing in the base number.

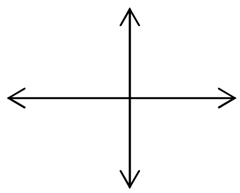
Another important logarithm is the natural logarithm with is base $e \approx 2.718281828$. We write this as $y = \ln x$ (which means $y = \log_e x$).

Now let's explore some translations. Graph the following on your calculator and sketch the graph. Here we will use $\log x$ to represent the common (base 10) logarithm

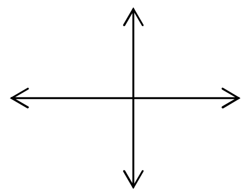
$y = \log x$



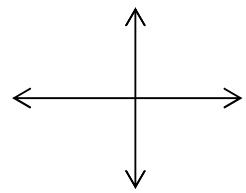
$y = \log x + 2$



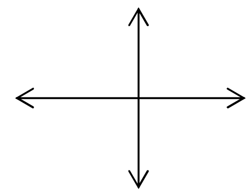
$y = \log x - 2$



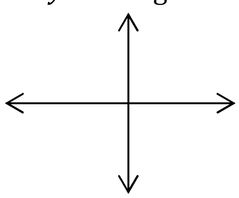
$y = \log(x + 2)$



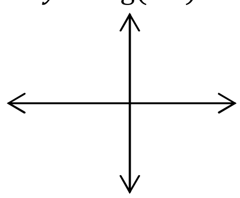
$y = \log(x - 2)$



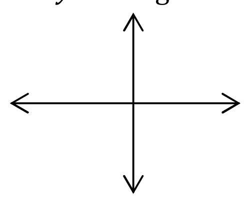
$y = -\log x$



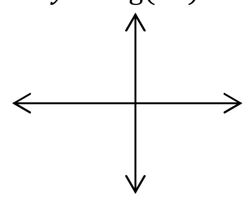
$y = \log(-x)$



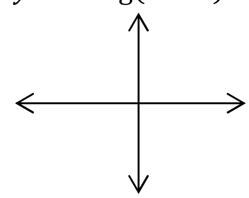
$y = 2\log x$



$y = \log(2x)$



$y = -\log(x + 2) - 2$



Describe how the values of a, b, c and d affect the graph of $y = a\log(bx + c) + d$

a:

b:

c:

d:

Finding the Domain of a Logarithmic Function

We discovered above that we cannot find the logarithm of a non-positive number (0 or negatives). We can use this idea to determine what the domain is of a logarithmic function by finding the set of numbers that forces the value inside the logarithm to be positive.

Try it: State the domain of each function.

a) $y = \log_3(x)$
b) $y = \log_5(x + 4)$
c) $y = \log(4x)$
d) $y = \ln(x - 5) + 10$
e) $y = 7\ln(2x + 5)$