Pre-Calculus Name: Date: Period:

4D: Problems with Logarithmic Functions

In this lesson we will use the properties of logarithms to solve equations that involve logarithms. We first need to recall these properties from earlier:

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$, and c a

Product Rule: $\log_b(PQ) = \log_b P + \log_b Q$

Quotient Rule: $\log_b\left(\frac{P}{Q}\right) = \log_b P - \log_b Q$

Power Rule: $\log_b(P^c) = c\log_b P$

Where b, R, and S be positive real numbers with $b \neq 1$, and c is any real number.

Change of Base Formula:

To convert $\log_b x$ into another base, use the formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

In practice, we use these:

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

We can use these properties to rewrite log expressions in different, more useful forms.

Example Assume that x and y are positive below. Write as a sum of logarithms with no exponents:

a) $\log\left(\frac{100x^2}{y}\right)$

b) $\ln\left(\frac{x^2y^3}{ez^5}\right)$

Example Write as a single logarithm:

a) $3\ln 2 - 2\ln 4 + \frac{1}{2}\ln 16$

b) $2\log a - (\log 2 + \log 5) + 3(\log b + \log 2c)$

Methods for solving Log Equations

To solve equations with logarithms, we can do one of the following:

1. Move all logarithms to one side, simplify to make one logarithm, then convert to exponential form and solve.
2. Write both sides of the equation as ***one*** logarithm with the same base, convert to exponential form, and solve.

   Note: ***Always check*** for problems with domain restrictions (the argument of a log must be positive).

After solving, you must check your domain to be sure that x is in the domain of the original function.

Example. Solve.

a) $\log_3(x+1) = 4$

b) $2\log x + 3\log 2 = \log 16$

c) $2\log_2 x + \log_2 4 = 3$

d) $\log_3(x+1) - \log_3(x+2) = \log_3 x$