Pre-Calculus Name: SOLUTIONS Date: Period:

2A.1: linear Functions

In this lesson, we will explore applications of the three primary forms of a linear function.

- **Slope-Intercept Form:** given the slope m of a line, and the y-intercept b of the line, the equation of the line can be given by

$$\boldsymbol{y = mx + b}$$

- **Point-Slope Form:** given the slope m of a line and a point (x_1, y_1) on a line, the equation of the line can be given by

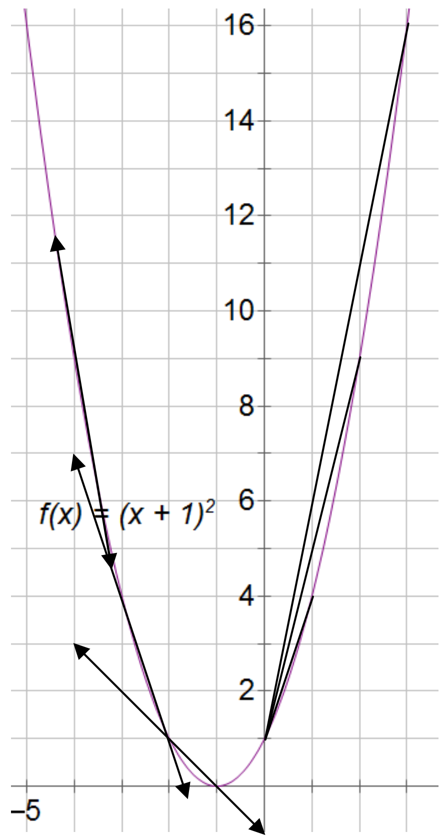
$$\boldsymbol{y - y_1 = m(x - x_1)}$$

- **Standard Form:** given information about the relationships between the variable x and y, we can often use the standard form

$$\boldsymbol{ax + by = c}$$

Part A: Equation of a secant line

The graph of the function $y = (x+1)^2$ is shown to the right.



1. We have previously learned that a secant line is a line that passes through two points on a graph. Find the slope-intercept form of the secant line to the $y = (x+1)^2$ curve for each of the following intervals:
   a. [0,1] $m = 3,\ y = 3x + 1$

   b. [0,2] $m = 4,\ y = 4x + 1$

   c. [0,3] $m = 5,\ y = 5x + 1$

2. Now, if we choose different endpoints for our interval, we will need to use the point-slope form of a linear equation.
   For each of the intervals below, find the point-slope for of the secant line to the $y = (x+1)^2$ line, then change this into the slope-intercept form.
   a. $[-2,-1]$ $m = -1, through\ (-1,0)$
   $y - 0 = -1(x+1)$
   $y = -1x - 1$

   b. $[-3,-2]$ $m = -3, through\ (-2,1)$
   $y - 1 = -3(x+2)$
   $y = -3x - 5$

   c. $[-4,-3]$ $\boldsymbol{m = -5, through\ (-3,4)}$
   $\boldsymbol{y - 4 = -5(x+3) \ \rightarrow \ y = -5x - 11}$

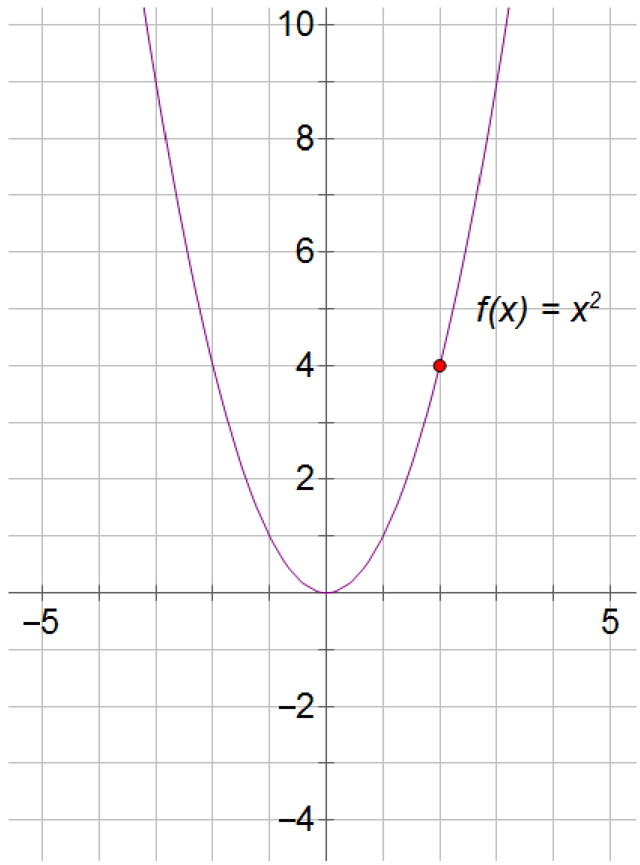
Part B: Equation of a tangent line

In a previous lesson, we have learned about the *difference quotient* which gives us an equation that determines the slope of a tangent line to a curve. For the function $y = x^2$, the difference quotient is

$$D(x) = 2x$$

This means that for the point (2,4) shown on the graph to the right, the slope of the tangent line is

$$D(2) = 2(2) = 4 = tangent\ slope\ at\ (2,4).$$



1. Since the slope of the tangent line at (2,4) is 4, write the equation of this line in point-slope form and graph it on the axis to the right.

   $y - 4 = 4(x - 2)$

2. Now let's find the line tangent to the curve at (1,1).
   a. Use the function $D(x) = 2x$ to find the slope of a line tangent to the curve at (1,1)

      $D(1) = 2(1)$

   b. Write the equation of the line tangent to $y = x^2$ at (1,1) and graph it above.

      $y - 1 = 2(x - 1)$
      $y = 2x - 1$

3. Now let's try two more lines tangent to the curve $y = x^2$. Repeat the steps in #2 to find the equation of the line tangent to $y = x^2$ at each of the points below. Then graph it above.
   a. $(-1,1)$
      $D(-1) = 2(-1) - 1 = -3$
      $y - 1 = -3(x + 1)$
      $y = -3x - 2$

   b. $(-3,9)$

      $D(-3) = 2(-3) = -6$
      $y - 9 = -6(x + 3)$
      $y = -6x - 9$

Part C: Related Rates Problems

Many situations involve totals of two rates or ratios that are working together or in opposition. For the situations below, write an equation that corresponds to the situation.

Two pipes are filling a 900 gallon tank with water. The first fills the tank at 10 gallons per minute, and the second fills it at 15 gallons per minute. The first pipe is turned on for x minutes and the second is turned on for y minutes.

1. Write an expression for the amount of water that the first pipe adds to the tank in x minutes.

$$10x$$

2. Write an expression for the amount of water that the second pipe adds to the tank in y minutes.

$$15y$$

3. Write an equation to model the possible times (x, y) that would completely fill the tank.

$$10x + 15y = 900$$

4.

5. If the first pipe is turned off and we only use the second pipe, how long will it take to fill the tank?

$$\frac{900}{10} = 90min.$$

6. If the second pipe is turned off and we only use the first pipe, how long will it take to fill the tank?

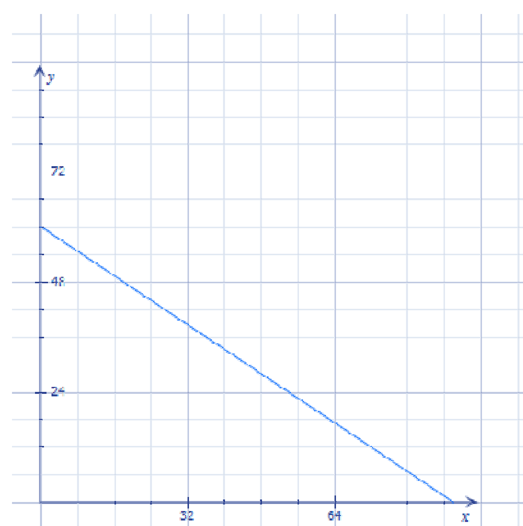
$$\frac{900}{15} = 60\ min.$$

7. What points on the graph of the equation in (a) do the times in (d) and (e) represent?

x and y intercepts

8. Draw the graph of the function in the first quadrant.

9. Explain what the points on this graph represent.



The points on the line are all the combinations of times that will fill the tank.