Pre-Calculus

Name: Date: Period:

2D: Piecewise Functions

A function is used to describe a relationship between two or more variables. However, sometimes this relationship changes for different input values. In this lesson, we will learn about a type of function called a ***piecewise function*** which is a way to combine parts of different functions into one function.

Example

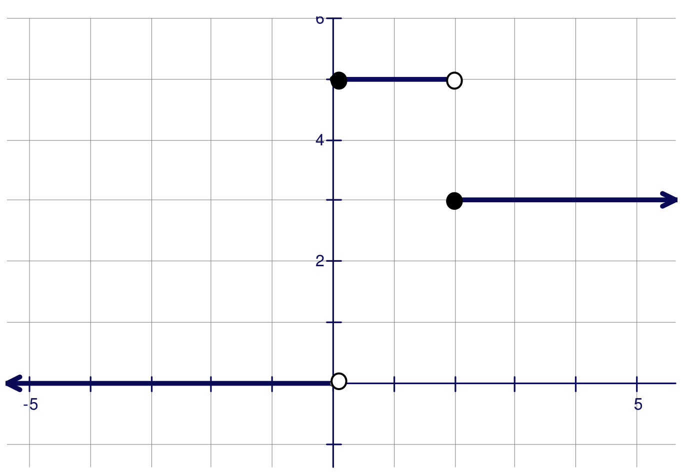
Suppose that an electrical engineer has designed an electronic circuit to use 0 volts before it is turned on. Once it is turned on it uses 5 volts for the first 2 seconds, then at exactly 2 seconds and after it only requires 3 volts to continue. Describe the voltage as a function of time and graph. Define $t = 0$ as the start time.

We will define this function over the domain $(-\infty, \infty)$ where negative numbers refer to time before the switch is turned on.
First, for time $t < 0$ we have 0 volts or $V(t) = 0$. Then for time $0 \leq t < 2$ the voltage is a constant 5 volts, so $V(t) = 5$ on this interval. Finally, for time $t \geq 2$, the voltage is a constant 3 volts, or V(t)=3.

In piecewise notation (or sometimes called the piecewise definition), we write

$$V(t) = \begin{cases} 0, & if\ x < 0 \\ 5, & if\ 0 \leq x < 2 \\ 3, & if\ x \geq 2 \end{cases}$$

Graphically, this function looks like this



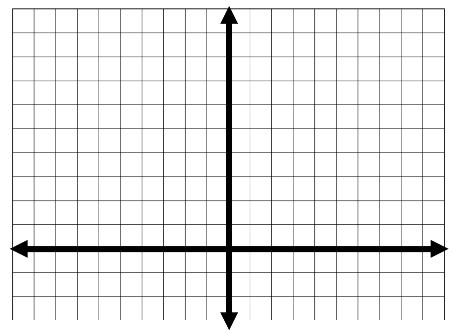
- The example above is called a ***piecewise constant function*** because the value of the function is constant at all intervals on the domain.
- Observe the endpoints of each segment or ray in the graph above. The endpoints that are not included in a certain interval are made open dots, while a closed dot denotes a point that is included in the interval (because the interval is defined by a <= or >=).
- We also note that the function is *discontinuous* since it has two points (x=0 and x=2) that have jump discontinuities.

Piecewise linear functions

A piecewise function definition like the one below gives a set of “rules” that define the output ($f(x)$) for any given input (x) value. Use the following function to answer questions 1 and2.

$$F(x) = \begin{cases} 0, & if\ x < 0 \\ x+1, & if\ 0 \le x < 5 \\ 2, & if\ x \ge 5 \end{cases}$$

1. Find
   $f(-2) =$
   $f(0) =$
   $f(1) =$
   $f(3) =$
   $f(5) =$
   $f(6) =$
2. Use these values to help you graph the function.



To plot piecewise functions, we can begin by graphing each of the functions (like $y = 0$, $y = x + 1$, and $y = 2$ above). Then “erase” part of the graph so that only part of the graph remains for the given domain piece. Endpoints are “closed” for included values and “open” for non-included values.

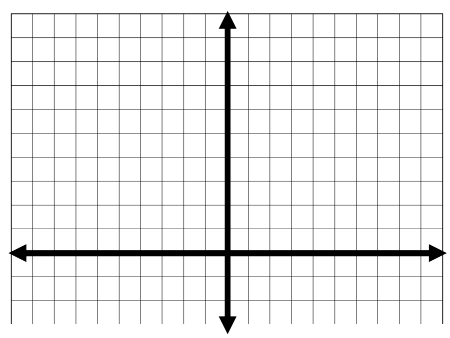
Mathematicians often choose to define more complicated functions using piecewise functions. This will greatly help us to model real-life situations

Try this

1. Plot this piecewise function

$$f(x) = \begin{cases} -x, & if\ x < 0 \\ x, & if x \ge 0 \end{cases}$$

2. What non-piece function has the same graph as this?

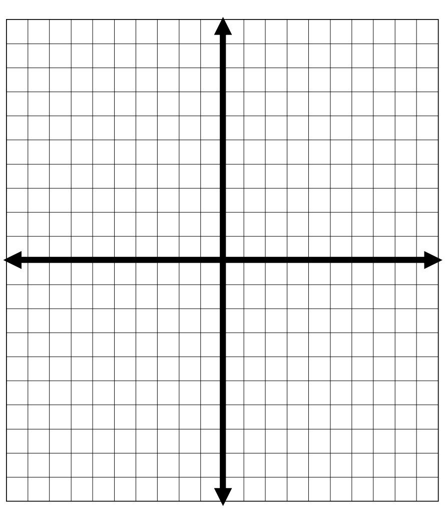


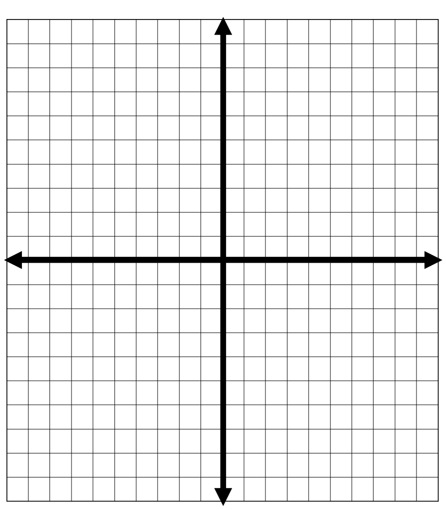
Try These Graph the piecewise functions and determine if they are continuous

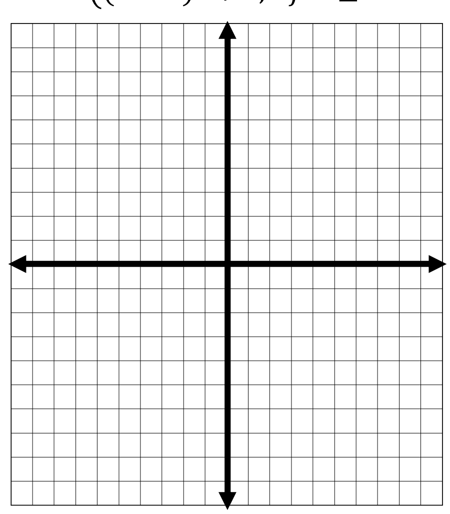
1. $y = \begin{cases} x-1, & if\ x \le 0 \\ x^2, & if\ x > 0 \end{cases}$

2. $y = \begin{cases} -2x+1, & if\ x \le 1 \\ -2x-1, & if\ x > 1 \end{cases}$

3. $y = \begin{cases} x+1, & if\ x < 0 \\ 1, & if\ 0 \le x < 1 \\ (x-1)^2+1, & if\ x \ge 1 \end{cases}$







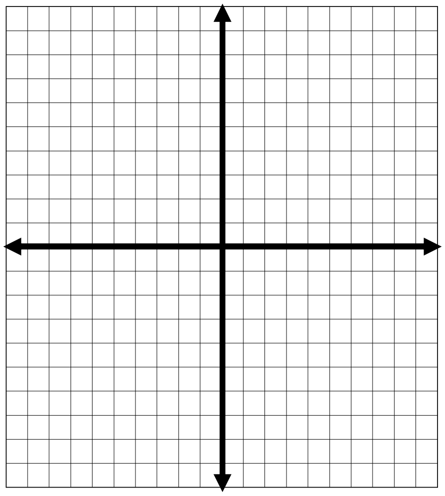
Assignment

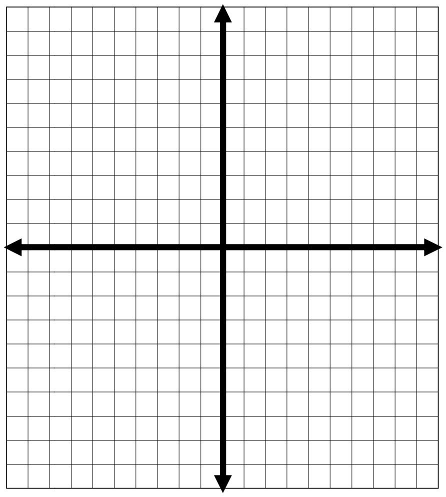
Graph the piecewise functions and determine if they are continuous

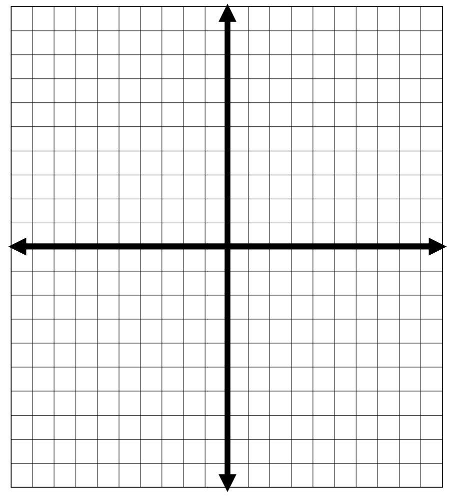
2. $y = \begin{cases} -\frac{1}{2}x, & if\ x \le 0 \\ \frac{1}{2}x^2, & if\ x > 0 \end{cases}$

3. $y = \begin{cases} 3x - 2, & if\ x \le 2 \\ 1, & if\ x > 2 \end{cases}$

4. $y = \begin{cases} [\![x]\!], & if\ x < 0 \\ x, & if\ 0 \le x < 1 \\ x^2, & if\ x \ge 1 \end{cases}$



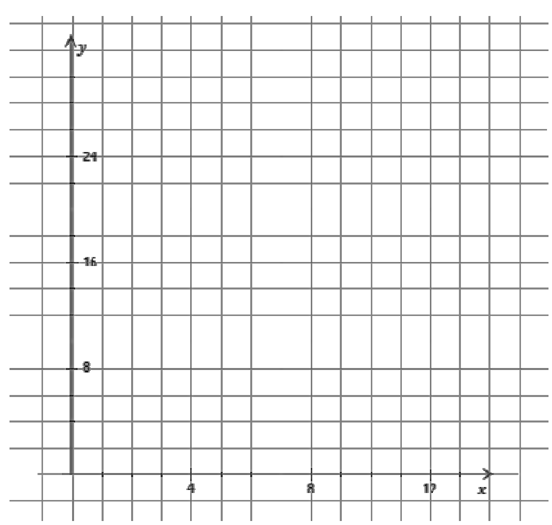




5. You are hired for a job, and your employer decides to pay you in the following way:
   - For the first year, your hourly wage will be \$10 while you are being trained.
   - On the first day of your second year of work, your hourly pay will be increased by \$4 for each year *or fraction of a year* that you have will worked for your employer.
     (For example, after working 1.25 years, your hourly wage will be $p = 10 + .25 \cdot 4 =$ \$11 per hour.)
   - When you \$30 per hour, you will no longer receive any raises.

   a) Write an equation for the hourly pay, p, as a function of the number of years, t, to describe each function and determine the domain represented by each part.

   b) Now graph the hourly pay as a function of the years employed.



   c) Write a piece function to describe this function with an overall domain of $(-\infty, \infty)$."

6. A skydiver jumps out of a plane with vertical speed of 0 ft/sec. His speed increases until he reaches a terminal velocity (i.e. maximum speed) of 176 ft/sec. The speed increases by the function $v(t) = \frac{1}{2}(32)t^2$. After he reaches his terminal velocity, his speed does not change. Write a piecwise function for the skydiver's velocity v in feet per second for time t in seconds. Then draw a graph for this function. (hint: you may want to find out how long it takes him to reach his terminal velocity first.