2E.1: Linear and Quadratic Modeling

Mathematical Modeling is the process of representing a relationship in algebraic terms using a combination of equations and graphs. Models can help us to better understand what is happening in system that we want to study.

Correlation is a measurement of how closely the model fits the actual relationship. Most models do not perfectly fit or predict the relationship that we are studying, but they help us see the big picture. Often we choose to make some assumptions or ignore some complications (such as ignoring wind resistance for a falling object, or ignoring discreteness) to make a useful model that gives us a good idea what is happening in the situation.

Modeling with Technology

When analyzing data, we can use technology to find a function that closely approximates the collected values. This process of going from specific points to an equation is called **regression**. In this lesson, we will be trying to find linear and quadratic equations that can be used to approximate a set of data. Below are the steps for using a TI-83 or TI-84 calculator to create a scatterplot and find a ***regression equation*** for the given data.

Steps for finding a regression equation:

Enter Data:

1. Go to [STAT] and select [1] to go to EDIT.
2. Enter the x-values into L_1 and enter the y-values into L_2.

Creating a scatterplot:

3. Now we need to graph these values in a scatterplot with the following steps.
4. Press [2nd] [Y=] to get the STAT PLOT menu.
5. Choose option [1] to get Plot 1
6. Select [On] to turn the plot on.
7. Choose the scatterplot option.
8. Since we used the lists L_1 and L_2., we do not need to change the Xlist or Ylist. If we did use a different list, we would need to change these.
9. To view the graph, hit the [ZOOM] button and choose ZoomStat which is option 9.

Finding a regression equation:

10. Now if we want find a linear regression, press [STAT] and choose CALC.
11. Choose the type of regression you want. For now, we will use LinReg and QuadReg for linear and quadratic regression.
12. This will send you back to the homescreen. You now just need to hit enter to get your equation.
13. (optional) If you want to store the equation in the equation editor, do this before pressing enter: Type a left-parenthesis, go to [Vars]→[Y-Vars] and select [Y1], type a right –parenthesis, then hit enter. Hit the [Y=] button to see your equation stored.

Try These: Now let's use the steps above to create a regression model for some data.

1. Make a scatter plot for the data below on your calculator and make a sketch of it here:

x	y
1	5
2	7.5
2.5	8
3	12
4	16
5	20

2. Find a linear regression model for the data and record it here:

   What is the "r" value given by your calculator?

   (Note: If your calculator doesn't give an "r" value, go to [Catalog] and scroll down until you see "Diagnostics On" and select this. Then repeat steps 10-13 in the instructions on the previous page. *The "r" value tells us how well the data fits the given equation*. The closer it is to 1, the better the fit is.)

3. Predict the y value when $x = 10$ with the linear model.

   This can be done manually, or by entering the equation into the equation editor, hit trace, then type in the x–value that you want.
   Or you can use [Vars]→[Y Vars] to select Y_1. Then type "$Y_1(10)$" into the homescreen and press enter.

4. Find quadratic regression model for the data and record it here:

   What is the "R" value given by your calculator?

5. Predict the y value when $x = 10$ with the quadratic model.

Assignment

For each of the data sets below,

a. Enter the data into your calculator and make a scatterplot.
b. Find a linear regression model, the associated "r" value, and make a prediction.
c. Find a quadratic regression model, and the associated "R" value, and make a prediction.
d. Then determine which model best fits the data.

1. The data below compares the number (x) of chirps/minute for the striped ground cricket and the temperature (y) in Fahrenheit

X (chirps)	Y (temp)
20	88.6
16	71.6
19.8	93.3
18.4	84.3
17.1	80.6
15.5	75.2
14.7	69.7
17.1	82
15.4	69.4
16.2	83.3
15	79.6
17.2	82.6
16	80.6
17	83.5
14.4	76.3

Reference: *The Song of Insects* by Dr.G.W. Pierce, Harvard

a. Scatterplot (rough sketch)

b. Linear regression equation:

r-value:

Use model to predict temperature for $x = 0$ chirps:

Use model to predict temperature for $x = 30$ chirps:

c. Quadratic regression equation:

r-value:

Use model to predict temperature for $x = 0$ chirps:

Use model to predict temperature for $x = 30$ chirps:

d. Which model fits best? Explain briefly.

2. The data below are the electricity consumptions in kilowatt-hours per month from ten houses and the areas in square feet of those houses

Home Size	KW Hrs/Month
1290	1182
1350	1172
1470	1264
1600	1493
1710	1571
1840	1711
1980	1804
2230	1840
2400	1956
2930	1954

McClave JT, Deitrich FH. Statistics. Macmillan 1991.

a. Scatterplot:

b. Linear regression equation:
r-value:

Use model to predict electricity consumption for a 500 square feet home.

Use model to predict electricity consumption for a 3500 square feet home.

c. Quadratic regression equation:

r-value:

Use model to predict electricity consumption for a 500 square feet home.

Use model to predict electricity consumption for a 3500 square feet home.

d. Which model fits best? Explain briefly.