4C: Problems with Logarithmic Functions

In this lesson we will use the properties of logarithms to solve equations that involve logarithms. We first need to recall these properties from earlier:

Properties of Logarithms
Let b, R, and S be positive real numbers with $b \neq 1$, and c a

Product Rule: $\log_b(PQ) = \log_b P + \log_b Q$

Quotient Rule: $\log_b \frac{P}{Q} = \log_b P - \log_b Q$

Power Rule: $\log_b P^c = c \log_b P$

Where b, R, and S be positive real numbers with $b \neq 1$, and c is any real number.

Example Assume that x and y are positive below.

a) Write as a sum of logarithms with no exponents: $\log \frac{3x^2}{y}$

$$\boldsymbol{\log 3 + 2\log x - \log y}$$

b) Write as a single logarithm: $3\ln 2 - 2\ln 4 + \frac{1}{2}\ln 16$

$$\boldsymbol{\ln 2^3 - \ln 4^2 + \ln 16^{1/2} = \ln\left(\frac{8}{16}\cdot 4\right) = \ln 2}$$

To solve equations with logarithms, we can do one of the following:

1. Set the equation equal to zero, graph the corresponding function, and find the zeros.
2. Write both sides of the equation as *one* logarithm with the same base, convert to exponential form, and solve.

After solving, you must check your domain to be sure that x is in the domain of the original function.

Example.Solve.

a) $\log_3(x+1) = 4$

$$\boldsymbol{x = 80}$$

b) $2\log x + 3\log 2 = \log 16$

$$\boldsymbol{\log(x^2 \cdot 2^3) = \log 16}$$
$$\boldsymbol{8x^2 = 16}$$
$$\boldsymbol{x^2 = 2}$$
$$\boldsymbol{x = \pm\sqrt{2}}$$

x cannot be negative in original problem! So the only solution is $x = \sqrt{2}$

Exercises

Assuming x and y are positive, use the properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms.

1. $\log_2 y^5$

   $\boldsymbol{5 \log_2 y}$

2. $\log_2 \frac{2x^3}{y^2}$

   $\log_2 2x^3 - \log_2 y^2$

   $\log_2 2 + 3\log_2 x - 2\log_2 y$

   $\boldsymbol{1 + 3\log_2 x - 2\log_2 y}$

3. $\log 1000x^4$

   $\log 1000 + \log x^4$

   $\boldsymbol{3 + 4\log x}$

Assuming $x, y,$ and z are positive, use properties of logarithms to write the expression as a single logarithm.

4. $\ln y - \ln 3$

   $\boldsymbol{\ln \frac{y}{3}}$

5. $4\log y - \log z$

   $\boldsymbol{\log \frac{y^4}{z}}$

6. $3\ln 2 - 2\ln 4$

   $\ln \frac{2^3}{4^2} = \boldsymbol{\ln \frac{1}{2}}$

   or ...

   $3\ln 2 - 2\ln 2^2$

   $3\ln 2 - 4\ln 2$

   $\boldsymbol{-\ln 2}$

Find the exact solution algebraically, obtain a numerical approximation, and check it by substituting into the original equation.

7. $\log_4(1 - x) = 1$

   $\boldsymbol{x = -3}$

8. $3\ln(x - 3) + 4 = 5$

   $\boldsymbol{x = \sqrt[3]{e} + 3}$

9. $3 - \log(x + 2) = 5$

   $\boldsymbol{x = -\frac{199}{100}}$

10. $\frac{1}{2}\ln(x + 3) - \ln x = 0$

    $\ln(x + 3)^{\frac{1}{2}} = \ln x$

    $(x + 3)^{\frac{1}{2}} = x$

    $x + 3 = x^2$

    $x = \frac{1 \pm \sqrt{13}}{2}$, by domain restrictions $\boldsymbol{x = \frac{1 + \sqrt{13}}{2}}$

11. $\log x - \frac{1}{2}\log(x + 4) = 1$

    $\log \frac{x}{\sqrt{x + 4}} = 1$

    $\frac{x}{\sqrt{x + 4}} = 10$

    $x = 10\sqrt{x + 4}$

    $x^2 = 100(x + 4)$

    $x^2 - 100x - 400 = 0$

    $x = 50 - 10\sqrt{29}$ *or* $x = 10\sqrt{29} + 50$

    Solution: $x = 10\sqrt{29} + 50$

12. $\ln(x - 3) + \ln(x + 4) = 3\ln 2$

    $\ln(x - 3)(x + 4) = \ln 2^3$

    $x^2 + x - 12 = 8$

    $x = -5$ *or* $x = 4$

    so .... $\boldsymbol{x = 4}$

13. $\log(x - 2) + \log(x + 5) = 2\log 3$

    $\log(x - 2)(x + 5) = \log 3^2$

    $x^2 + 3x - 10 = 9$

    $x^2 + 3x - 19 = 0$

    $x = \frac{-\sqrt{85} - 3}{2}$ *or* $x = \frac{\sqrt{85} - 3}{2}$

    Solution: $\boldsymbol{x = \frac{-3 + \sqrt{85}}{2}}$

14. Determine whether a linear, logarithmic, exponential, power, or logistic regression equation is the best model for the data using your calculator and finding the R^2 value.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Alaska's Population	63.6	64.4	55.0	59.2	72.5	128.6	226.2	302.6	401.9	550.0	626.9

$\boldsymbol{Best\ Curve = Logistic}$