

Unit 4 Test Review

Learning Targets: 4A-4C

Complete the problems below, show your work, and write your answer in the blank provided.



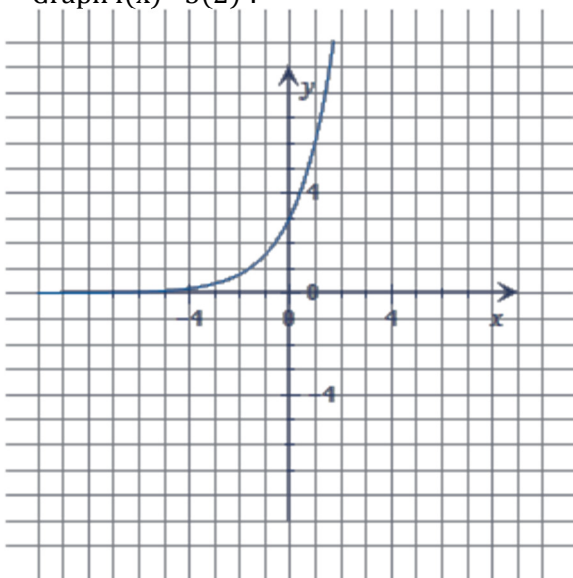
Target 4A

I can graph and describe transformations for exponential and logarithmic functions.

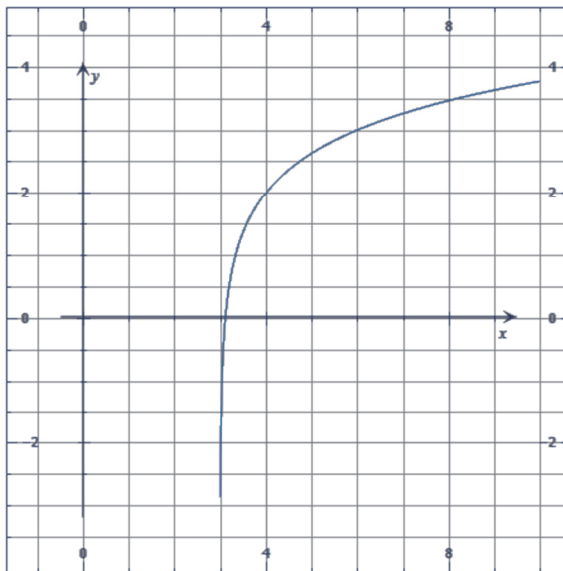
- Describe the transformations that change $f(x) = \log_2 x$ to $g(x) = 3 + 2\log_2(x-3)$.

Up 3 by the first 3, Right 3 by $(x - 3)$, and Stretched vertically by a factor of 2

- Graph $f(x) = 3(2)^x$.



- Graph $f(x) = 2 + \log_3(x-3)$



- Write the equation for $f(x) = 4^x$ that undergoes the transformations of being shifted 2 units right, 5 units down, and reflected across the y-axis.

$y = 4^{-(x-2)} - 5$



Target 4B

I can solve problems involving exponential or logistic functions.

5. Solve $3(8^x) = 50$ for x . Round your answer to the nearest tenth.

$$8^x = \frac{50}{3}$$
$$x = \log_8\left(\frac{50}{3}\right) = 1.353$$

6. Given the function $f(x) = 3(0.5)^x$, does $f(x)$ represent exponential growth or decay? Explain how we know.

Exponential Decay since the base, 0.5, is less than 1.

7. Given the function $f(x) = 12 / (1 + 3(0.2)^x)$

- a. What is the limit of growth?

$$\text{Limit: } y = 12$$

- b. What is the y-intercept of $f(x)$?

$$\text{Let } x = 0$$
$$y = \frac{12}{1 + 3(0.2)^0} = \frac{12}{1 + 3} = \frac{12}{4} = 3$$
$$y - \text{intercept: } (0, 3)$$

8. Write the exponential function that passes through the points (0, 5) and (4, 405).

$$y = a(b^x)$$
$$\text{"initial value"} = a = 5$$

$$\text{Since } 5 = a(b^0) \rightarrow 5 = a$$

So, $y = 5(b^x)$. Now substitute (4, 405)

$$405 = 5(b^4)$$
$$81 = b^4$$
$$\sqrt[4]{81} = b$$
$$3 = b$$

$$\text{Function: } y = 5(3^x)$$

Target 4C*I can solve problems involving logarithmic functions.*

9. Solve $\log_3 \sqrt{x-2} = 2$ for x .

$$\begin{aligned}\sqrt{x-2} &= 3^2 \\ (\sqrt{x-2})^2 &= (9)^2 \\ x-2 &= 81 \\ x &= 83\end{aligned}$$

10. Evaluate $\log_4 12$.

$$\log_4 12 = \frac{\log 12}{\log 4} = 1.792$$

11. Write the expression as a single logarithm.

$$2\log_3 X + 4\log_3 Y - 3\log_3 Z$$

$$\begin{aligned}\log_3 x^2 + \log_3 y^4 - \log_3 z^3 \\ \log_3 x^2 y^4 - \log_3 z^3 \\ \log_3 \left(\frac{x^2 y^4}{z^3} \right)\end{aligned}$$

12. Write the expression as the sum or difference of logarithms.

$$\log_2(x^2/(a^2b^3))$$

$$\begin{aligned}\log_2 \left(\frac{x^2}{a^2 b^3} \right) &= \log_2 x^2 - \log_2(a^2 b^3) \\ &= \log_2 x^2 - (\log_2 a^2 + \log_2 b^3) \\ &= 2 \log_2 x - 2 \log_2 a - 3 \log_2 b\end{aligned}$$

Applications

13. A telescope is limited in its usefulness by the brightness of the star it is aimed at and by the diameter of its lens. A formula for the limiting magnitude L of a telescope, that is, the magnitude of the dimmest star that it can be used to view, is given by

$$L(d) = 9 + 5.1 \log(d)$$

Where d is the diameter (in inches) of the lens.

- a) State the domain of this function.

$$\text{Domain: } (0, \infty)$$

- b) What is the limiting magnitude of a 3.5-inch telescope?

$$L(3.5) = 9 + 5.1 \log(3.5) = 11.775$$

- c) What diameter is required to view a star of magnitude 14?

$$14 = 9 + 5.1 \log(d)$$

$$5 = 5.1 \log(d)$$

$$\frac{5}{5.1} = \log(d)$$

$$d = 10^{\frac{5}{5.1}} = 9.559$$

14. Calculate the number of years necessary for \$250 to grow to \$750 at 4.3% compounded continuously. Use the compound interest formula: $A = Pe^{rt}$, where A = final amount, P = starting amount, r = interest rate, and t = time in years. Show your work and round your answer to the nearest tenth.

$$750 = 250e^{.043t}$$

$$3 = e^{.043t}$$

$$\ln 3 = .043t$$

$$\frac{\ln 3}{.043} = t$$

$$25.549 = t$$

15. Use the data in the table below.

x	0.25	0.5	2	4	8	15
y	-2.52	-1.38	1.45	2.18	4.15	5.91

- a. Write a natural logarithmic function for the data.

$$y = .03485 + 2.001 \ln(x)$$

- b. What is the value of the function when $x = 20$?

$$y = 6.03001$$