1B: Identifying Functions

I. Relation vs. Function

Relation – Any set of ____________________ ____________________

Example 1: Average Gross Monthly Salaries=
{(Physician, \$11,698),(Airline Pilot, \$5,884), (Computer Programmer, \$5,378), (Salesperson, \$2,260), (furniture finisher, \$1,977)}

Domain – The set of all ____________________ components in a relation. (a.k.a. *x-values*)

Range – The set of all ____________________ components in a relation. (a.k.a. *y-values*)

Example 2:

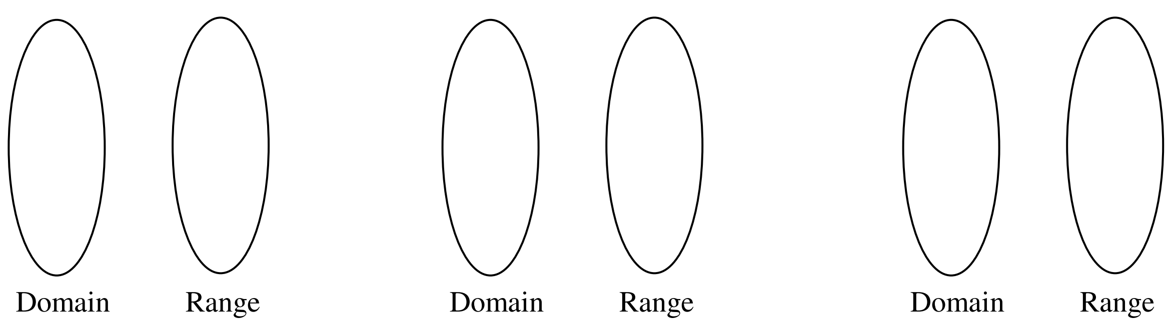
Salary Domain =

Salary Range =

Mapping: A relation can be mapped to show how the domain is connected to the range.

Example 2: *Draw a map for these relations.*

a. {(2,0), (4,2),(5,1),(10,12)} b. {(3,4),(5,4),(6,-1),(7,5)} c. {(3,2),(3,9),(4,6),(5,9)}



Functions

Some relations are unique because they define one specific outcome for every domain element.

Example 3: Which of the following statements are *always* true?

a. A person's height is determined by their age.
b. An hourly worker's paycheck is determined by the hours they work.
c. A person's vision is determined by the amount of T.V. they watch.
d. The distance a car drives on the freeway (at the speed limit) is determined by the amount of time it drives.
e. The number of assigned problems and the time needed to complete them.

Which of these statements describe functions?

> ***Definition:*** *A* ***function*** *is a relation such that*
>
> *for every __________________(x) value, there is only one _______________(y) value*

II. Function as equations and graphs

Every function must have ***independent*** variable(s) which determine the value of the ***dependent*** variable.

You are familiar with functions with one dependent variable, like:

$$y = \frac{2}{3}x + 4, \qquad y = 3x^2 + 4 - 5, \qquad f(x) = 2\sin(x)\,.$$

However, we can also have functions of more than one variable:

$$z = 2x + 3y + 4, \qquad f(x, y) = \sqrt{x^2 + y^2},$$
$$f(x, y) = -(\sin \pi x)(\cos \pi x) + \sin(4\pi x)\sin(4\pi y)$$

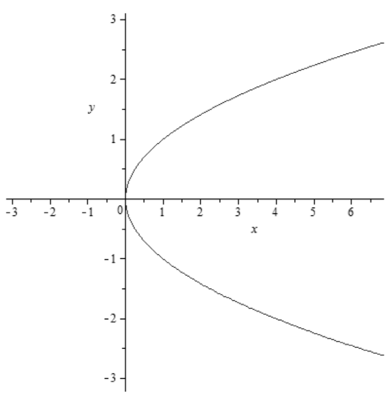
Example 4: Which of the equations below determine y as a function of x

a) $y = \sqrt{4x + 3}$
b) $y = 3x + 4z$
c) $0 = x - y^2$
d) $9 = 3xy$

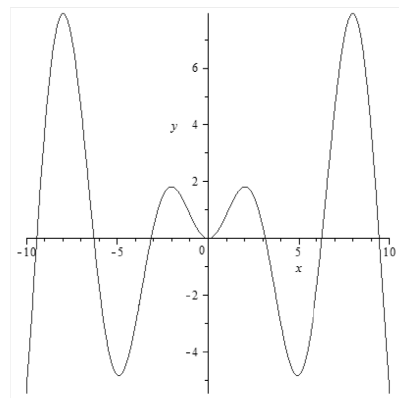
Vertical Line test – if a graph represents a function, then there are no vertical lines that can intersect the graph more than once.

Example 5: Which ones of the following graphs represent functions.

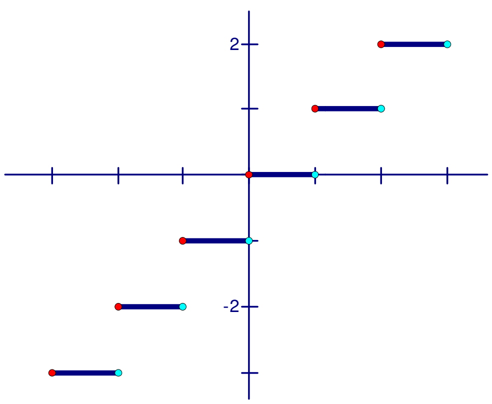
a.



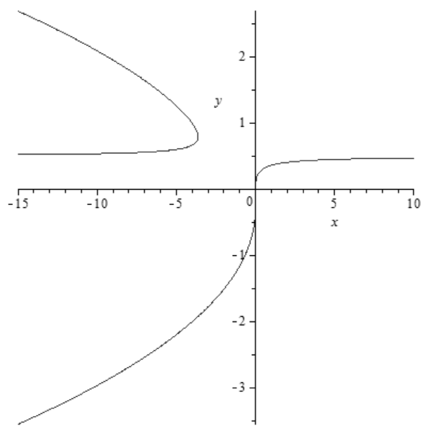
b.



c.



d.

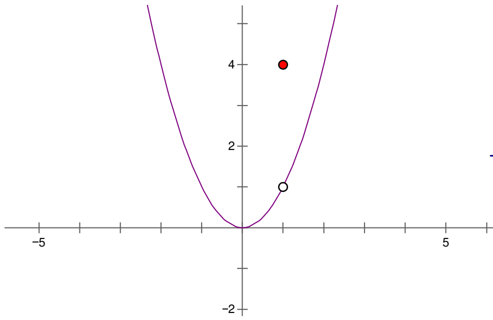


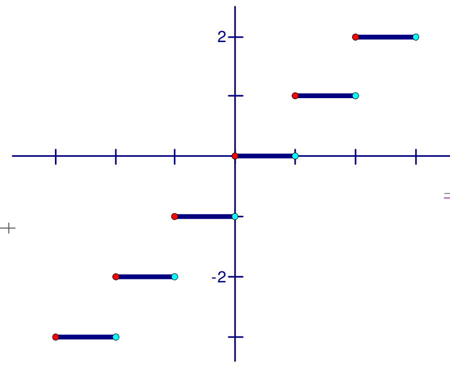
III. Continuity

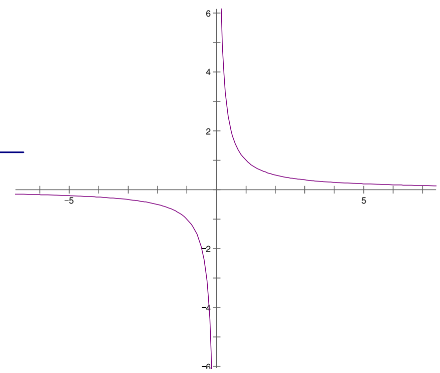
A function is **continuous** if for all values in an interval if for all values of a in the interval, $f(a)$ is where it is expected to be to make a "smooth" curve. Otherwise the function is **discontinuous** on the interval. Graph (b) in the previous example is *continuous.*

Types of discontinuity

Removable discontinuity *Jump discontinuity* *Infinite discontinuity*







Try These

Predict the domain and range and describe it in interval notation, then verify it with your graphing calculator. Also, use the graph to decide what type of continuity it has.

Remember: *radicands must be non-negative, and denominators must be non-zero!*

Function	Domain	Range	Continuity
$y = x^2$			
$y = x^2 - 8$			
$y = \sqrt{x}$			
$y = \sqrt{x - 5}$			
$y = \sqrt{5 - x}$			
$y = \frac{1}{x}$			
$y = \frac{1}{x + 1}$			
$y = \frac{\sqrt{1 - x}}{x - 5}$			
$y = \frac{1}{\sqrt{x}}$			
$y = \frac{1}{\sqrt{x^2}}$			
$y = \frac{1}{\sqrt{x^2 - 4}}$			
$y = \frac{1}{\sqrt{4 - x^2}}$			
$y = \frac{1}{\sqrt{4 + x^2}}$			