

Engaging Math with Geometer's Sketchpad

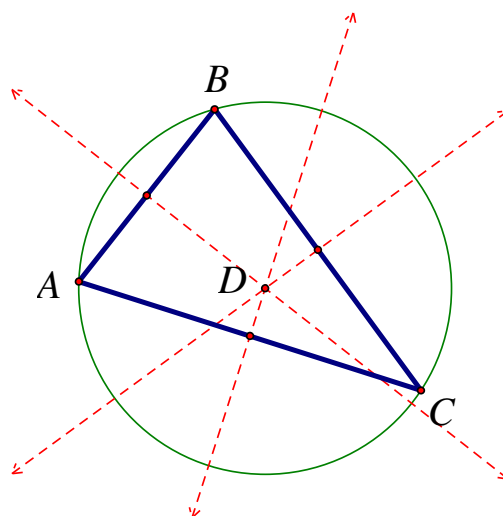
Geometer's Sketchpad (GSP) by Key Curriculum Press is a powerful tool for making Algebra and Geometry come alive. This software extremely effective when used as a *demonstration* tool in class presentations, or as a *discovery* tool that engages students in deep mathematical reasoning or as a *design tool* for printed material.

I. GSP as Demonstration Tool

Back in the day of dusty chalkboards, mathematical figures were stuck on the board (until the teacher accidentally wiped it off with their sleeve!) leaving students to try to use their imagination as they ponder the possible outcomes and implications of the concept. Today, we have the technology available to make these figures come alive to help students fully understand deep mathematical concepts.

Geometers Sketchpad is a perfect tool for teachers of all age students to make geometric and algebraic concepts able to be learned in a more profound and lasting way. The following are a few examples of ways to use GSP as a demonstration tool:

1. Model problems like the "Overlapping square" (Overlapping squares.gsp.)
2. Model the Pythagorean theorem (ExploringThePythagoreanTheorem.gsp)
3. π race (PiRace.gsp)
4. Graphing Quadrilaterals (QuadraticExploration.gsp)
5. Demonstrate Slope-intercept form
6. Completing the square (square-trinomial.gsp)
7. Multiplying fractions - unit square broken into two fractional parts



II. GSP as Discovery Tool

One of the most important applications of Geometers Sketchpad is as a tool to engage students in *deep mathematical reasoning* through exploration and discovery. When students are given the opportunity to construct and manipulate geometric and algebraic figures, mathematics becomes tangible and the level of comprehension can greatly improve.

GSP activities designed for students can either be done with students following instructions to create their own sketches, or they can manipulate pre-made sketches to discover new concepts or model what they have learned in the classroom.

It's important to note that the *Common Core State Standards* and *Smarter Balance Assessment* will incorporate many dynamic geometry and algebra problems incorporating technology based on software similar to Geometer's Sketchpad (see last page). So, this is a skill students need to learn!

Student created sketches

These types of activities allow students to discover and practice mathematical concepts while being directed by the teacher. All of these activities begin with a blank sketch.

Important Note: Students should be given enough instruction to direct them to the desired discovery, but don't give too specific instructions! As much as possible, keep instructions general so students can learn by grappling with mathematical figures.

** Remember, a geometric figure (like a square) is only correct if the desired characteristics remain when any point or line is moved.*

1. *GSP Introduction activity.*

This handout is a good way for students to learn how to use sketchpad.

2. *Exploring and Discovering*

Try these geometric constructions:

- a) Prove that the sum of the angles in a triangle is 180°
(use the [calculate] tool to show a sum.)
- b) Construct an isosceles triangle and an equilateral triangle.
- c) Construct a Parallelogram, rectangle, and a square.
- d) Translate a Triangle
 - i. Graph the triangle $A(1,2), B(2,5), C(4,3)$ (label points)
 - ii. Select the y-axis select [Transform]→[Mark Mirror]

3. *Algebra - graphing functions*

a) *Graphing Lines and Discovering Equations.*

- i. Turn the grid on using the [Graph] menu.
- ii. Choose [Snap Points], and draw a line.
- iii. Select the line and choose [Measure] → [Equation]
- iv. Now move the points and explore slope-intercept form.

b) *Graphing higher degree functions.*

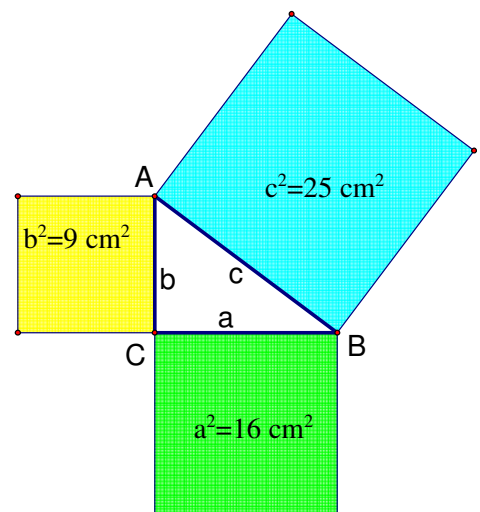
- i. Select [Graph]→[Plot Function]
- ii. Enter a polynomial or other function (no "y=" needed).
- iii. Repeat as many times as you want to compare functions.

Teacher created activity

These type of activities allow students to manipulate a pre-made sketch to make discoveries.

1. Pythagorean theorem
(*Exploring the Converse of the Pythagorean Theorem.gsp*)
2. Angry Birds Parabolas
(*AngryParabolasSketch.gsp*)
3. Others:

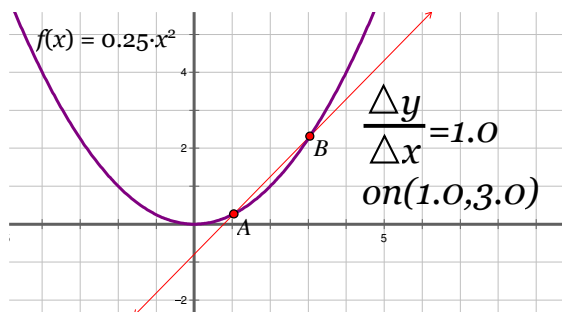
Most of the demonstration sketches in Part I could also be used by students to get a hands-on experience with their math.



III. GSP as a Design Tool

“Figures not drawn to scale” can either mean that we want students to understand a general concept, or it means we couldn’t find a better graphic to use! GSP is a great way to make quick and accurate mathematical images for class handouts and assessments. Furthermore, since GSP sketches are created with variable positions, creating figures in GSP makes it easy to slightly change the figure and have a *second version* for a different problem or 2nd version of an assessment.

As examples, all of the graphics in the body of this handout were created on Geometer’s Sketchpad. To copy an image from GSP to a word processor document, just select the parts you want to copy, or use “Select All” in GSP, then paste it into your document. It’s as easy as that. You can then resize and crop the final image as needed.



Try creating images and for the following problem types and copy the image into a document.

1. “Write the equation of this line in slope intercept form.”
Use [Show Grid], [Snap Points], and graph a line with the line tool.
2. “Find the area of this triangle.”
Construct a triangle, Construct the altitude (hide the ppd line, show a segment), Measure the length of the base, and Measure the length of the altitude.
3. “Find the measure of the missing angle.”
Construct a triangle, measure two angles (or 1 if its isosceles), then copy and paste.
4. “Write the equation of this parabola in standard form”
Use [Graph]→[Plot New Function] to enter the desired equation, choose “Select all”, de-select the equation, then copy and paste.
5. Make up your own question to match a figure to.

IV. Math Illustrations

While GSP is a great tool as we have seen above, it is somewhat limited in annotation ability, although GSP version 5 does have some improved tools for annotation. When designing figures for print, one alternative program is Math Illustrations (www.mathillustrations.com). This program allows you to do the same types of constructions as GSP, but also has the ability to easily mark angles, sides, etc. for print graphics. Another nice feature is the ability to set angles and parameters.

Try constructing and annotating these figures in *Math Illustrations*:

1. Isosceles triangle with marks
2. Parallelogram with marks
3. 30-60-90 triangle

V. Links

- Geometers Sketchpad Home: <http://www.dynamicgeometry.com/>
- Geometers Sketchpad Classroom Activities to download: <http://sketchexchange.keypress.com/>
- Math Illustrations: www.mathillustrations.com

VI. Some Common Core State Standards Addressed

Here is *just a sampling* a few places where software like Geometer's Sketchpad is directly applied to the Common Core State Standards

- Geometry 7.G:
Draw, construct, and describe geometrical figures and describe the relationships between them.
 - Draw (freehand, with ruler and protractor, and with **technology**) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- Geometry 8.G:
*Understand congruence and similarity using physical models, transparencies, or **geometry software**.*
 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
 - Understand and apply the Pythagorean Theorem.
 - Explain a proof of the Pythagorean Theorem and its converse.
- H.S. Algebra A-SSE: Seeing Structure in Expressions
Write expressions in equivalent forms to solve problems
 - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
 - Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- H.S. Functions F-IF: Interpreting Functions
Interpret functions that arise in applications in terms of the context
 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
 - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. *Estimate the rate of change from a graph.*
 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and **using technology** for more complicated cases
- H.S. Geometry G-CO: Congruence
Experiment with transformations in the plane
 - 2. Represent transformations in the plane using, e.g., transparencies and **geometry software**; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
Make geometric constructions
 - 12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, **dynamic geometric software**, etc.).