Hands-on

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Brain-on!

"I hear and I forget, I see and I remember, I do and I understand."

- Chinese proverb

Manipulatives and "hands-on" activities can be the key to creating concrete mathematical understanding for students of all levels. Not all students can soak in a verbal lecture and master a concept by merely hearing or even seeing a demonstration. Most students need the opportunity to wrestle and explore a concept in a more tangible way. Manipulatives are the perfect vehicle for providing students with this invaluable opportunity.

Many commercially produced manipulatives are available through educational supplies (see below for some), but don't be afraid to think outside the box. Many common items can be used for mathematical exploration and modeling (and they can save a lot of money). So, be creative and keep your students hands and brains actively learning.

Ways to use Manipulatives and hands-on approaches in Mathematics classes:

- Demonstration ("I see and I remember")
- Exploration ("I do and I understand")
- Practice and real-world application
- And just to have Fun!

Remember: Using manipulatives and hands-on activities can take time…
but the return on your investment is exponential!

Puzzles, Patterns, and Plastic Models

Getting students to get their hands "dirty" as they use mathematical thinking will build a strong foundation that will help students to truly understand a concept and be more likely to be able to apply it in the future. Let's look at a few examples of activities that can model this concept. Here are some notes that may be helpful as you consider the activities (in the back of this packet).

I. Puzzles: Squares, Triangles, and Stretching Circles

Puzzles encourage students to be curious. A good puzzle can develop problem solving and multiple content strands and be fun at the same time.

A. **5-Square Puzzle:**

This is a good puzzle to promote problem solving, fun with math, and the practical use for simplifying radicals.

1. Use the pieces from the 5 squares to make one large square.
2. Having Trouble? Consider the areas. The area of the whole=the area of the parts
   What does the length of the side need to be?
   Now find a side of the pieces that can give you the side length needed.

B. **A Postal Packing Problem:**

This puzzle takes an interesting twist on the relationship between volume and surface area. A shipping company needs to box up six $1 \times 2 \times 2$ packages into one large box.

1. What are some possible size boxes that could be used to pack all six packages together?

2. If the postal company wants to use a box with the least surface area, what should the dimensions be?

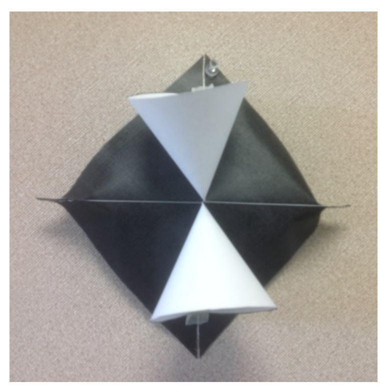
3. What if you add three more smaller $1 \times 1 \times 1$ packages? Can you find the dimensions of the box with the least surface area?

C. **Slicing Cones and Spinning Ellipses**

Some hands-on activities can be used to engage students for an entire lesson or more, while others can be a quick, simple, and effective way to solidify concepts in student's minds and help them visualize concepts better.

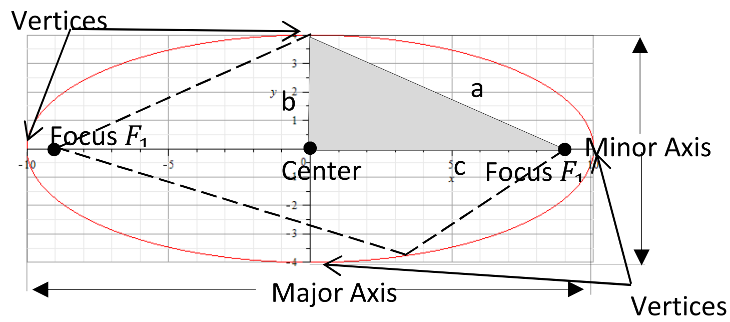
Make your own Double Cone:

When working with conic sections, students are asked to visualize a plane intersecting two cones that share a vertex and have collinear heights! That's a lot to visualize. The Double-Cone Maker is a fun way to give students a tangible way to discover the intersections of a plane and a double-cone.



Make your own ellipse:

When teaching students that an Ellipse is the set of all points whose distances to two foci have a constant sum, we could just state the definition, or show a picture. However, if we can get students to create their own ellipses, they will be more likely to have a better grasp on the figure.



To make your own ellipse, all you need is a set of softwood boards or cardboard (about 5”x5”), push pins, string, and photocopied coordinate grids. Give each student a board, 2 push pins, an 8” string tied in a loop, and a page of grids. Then allow students to put pins in different positions as the foci and draw their own Ellipse. Want to do a larger version of this, try using ropes and chalk out on a blacktop to make large ellipses with given values of major and minor axes.

II. Patterns and “The Wishful Thinking Method”

Patterns are a

A. **Hands-on Exploration Problem: “Leap Frog”**

1. Try to solve the Leap frog puzzle with basic (backwards moves allowed) and challenging (no backwards moves allowed) rules.

2. “What is the minimum number of moves?”

3. What is the minimum number of moves if we had 20 pegs on each side?

   This can be a tough question. Try using the Wishful thinking method:

B. The ***Wishful Thinking Method*** (George Polya) is a technique that engages students in active learning and allows them to be construction crew of their own understanding (If they build their knowledge, they will remember and understand it better).

Step 1: Wish for an easier Problem

For the Leap Frog puzzle, wish that there were only two pegs.

Step 2: Use inductive reasoning to find a pattern that answers the original problem.

For the Leap Frog Puzzle, Keep adding pegs to find a pattern.

Inductive reasoning puts students in the driver seat of their mathematical comprehension. When students learn to use the Wishful Thinking Method and inductive reasoning, they are empowered them to perpetuate their Mathematical understanding from one concept to the next.

It's been said: "Give a man a fish and you'll feed him for a day. Teach him to fish, and you'll feed him for a lifetime." Inductive reasoning is the power to fish. We must teach them to fish!

III. Plastic Models

Using manipulatives for demonstration can be quite powerful. Many students have a difficult time "seeing" abstract concepts, so make it concrete for them.

A. Quick Counting
   1. The Greeks saw numbers geometrically. Teaching students to see number patterns this way can be very powerful. Three number families are very useful to "picture":
      Square Numbers
      Oblong Numbers
      Triangular Numbers
   2. Problems in the Quick Counting handout are good examples of the applications of these figurative numbers

B. Golf Ball Models
   Having a few of these models on hand can be very useful for demonstration. The first three models below are ones I use most

   Most useful:
   1. Triangular numbers = sum of counting numbers
   2. Square numbers = sum of odd numbers
   3. Sum of 2 triangular numbers = oblong numbers

   Other good models:
   4. Extra Oblong Models
   5. Sum of triangular numbers = Tetrahedral numbers

IV. A Few More Ideas:

The activities above and more can be found at my website: www.knightmath.com .

Here are just a few ideas of manipulatives that you can use for different content strands (not necessarily limited to a specific course)

Algebraic Reasoning

- *Blocks:* Wooden blocks and/or cm blocks are vir
- *Chips*: Colored plastic chips give a great visual model for integers, showing patterns, and representing figurative numbers (square numbers, triangular numbers, oblong numbers, etc.)
- *Algebra Tiles*: Model variable expressions with incommensurable tiles. Very useful for modeling many Algebraic properties in a tactile activity.
(Online Version: at http://nlvm.usu.edu/)
- *Chart Grid Paper*: Use colored dots to collect class data to analyze functions and inequalities.
- *Playing Cards*:
    - Modeling and practicing integers: Let reds be negative and blacks be positive. Draw several cards and find sum. Try making it a game with multiple players.
    - Modeling expressions, equations and like terms (example: ignore the card numbers and we have $3b + 4r = 1b + 3r + 2b + 1r$. Give a value to black cards and a value to red cards and evaluate both expressions to verify the validity of "Combining like terms")
- *Mirrors*: Many ways to use mirrors for similarity and proportional reasoning. Don't just talk about it…. Take time to do it!
- *Miras*: Semi-opaque tool for drawing perpendicular lines. Great for discovering slopes of perpendicular lines.
- *Graphing Calculators*: invaluable for clearly seeing linear applications. Excellent for curve fitting using the [STAT PLOT] on a TI-83/84.

Geometry:

- *Solid Models*: Plastic models available for demonstrating prisms and pyramids and comparing volume and surface area.
- *Blocks*: Wooden blocks, 1-cm cubes, and inter-locking "Hex-A-Link" cubes are very useful for many volume/surface area explorations.
- *Polygon Sticks*: Commercially available sticks and connectors for making polyhedral.
- *Drinking Straws ,Bamboo Skewers, Toothpicks*: A cheap way to model polygons and polyhedral. They can be taped together or skewers can be stuck into marshmallows
- *Paper, scissors, and tape*: not fancy manipulatives; however, almost any geometric theorem or postulate can be modeled with these things. A great curriculum that models this is "Discovering Geometry" by *Key Curriculum Press.*

- *Patty Paper*: Can be used to fold and cut to demonstrate Geometric theorems. Literature is available.
- *Miras*: Semi-opaque plastic tool for drawing reflections.
- *Geometer's Sketchpad*: An amazing tool for Geometry. Every Geometry teacher should have and use this. See below.

Trigonometry:

- *Clinometer*: Excellent activity
- *Geometer's Sketchpad*: Very useful for easily showing and creating dynamic demonstrations for topics like the Unit Circle

Probability:

- *Dice, coins, spinners*: Endless hands-on opportunities. Experiment first, then get theoretical.
- *TI-83/84 Applets*: Random number generators, and virtual probability applet "ProbSim"

Technology:

- *Graphing Calculators:*
  - Seeing graphical patterns quickly and dynamically
  - Programming simple programs to use and understand variables and explore a real-world application.
    Try this with students to make the calculator count:

    ```
    Program: COUNT
    :ClrHome
    :Lbl 1
    :1×C
    :Disp C
    :C+1×C
    :Goto 1
    ```

    You can use simple programs like this to talk about the importance of variables when working with computers.
- *Virtual Manipulatives and online applets/activities:*
  - National Library of Virtual Manipulatives: http://nlvm.usu.edu/
  - NCTM Illuminations: http://illuminations.nctm.org/
- *Geometer's Sketchpad:* A "must-have" and a "must-use" for any Geometry teacher as well as algebra teacher. This is available from *Key Curriculum Press*. When Sketchpad is used with a *Smartboard*, students can't help but pay attention to the moving polygons.
- *Vernier Sensors*: These sensors can be plugged into a TI-84 to measure variables such as motion and temperature. These are excellent tools for activities on Distance/Rate/Time and analyzing real-world data with linear and non-linear functions.
- *TI-SmartView*: This is the latest Graphing Calculator Emulator from Texas Instrument. It works great when projected in front of a class. In conjunction with a smartboard, it makes a 3-foot tall graphing calculator with a huge-display!