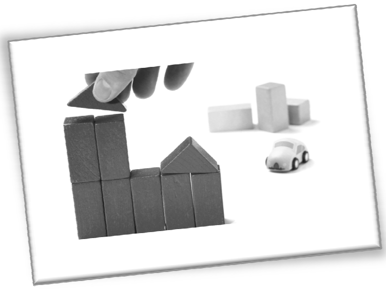
Hands-on



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Brain-on!

A Trio of Puzzles

"I hear and I forget, I see and I remember, I do and I understand."

- Chinese proverb

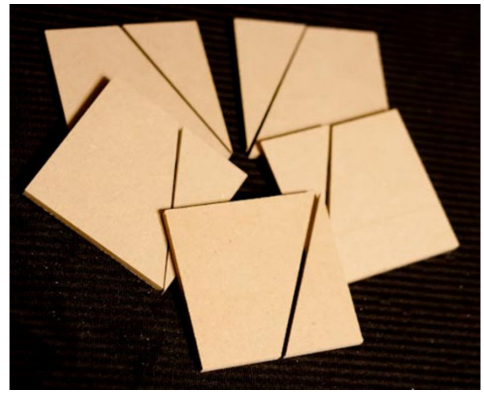
Manipulatives and "hands-on" activities can be the key to creating concrete mathematical understanding for students of all levels. Not all students can soak in a verbal lecture and master a concept by merely hearing or even seeing a demonstration. Most students need the opportunity to wrestle and explore a concept in a more tangible way. Manipulatives are an excellent vehicle for providing students with this type invaluable opportunity.

Many commercially produced manipulatives are available through educational supplies, but don't be afraid to think outside the box. Many common items can be used for mathematical exploration and modeling (and they can save a lot of money). So, be creative and keep your students hands and brains actively learning. When used well, hands-on approaches can be used as

- Demonstration ("I see and I remember")
- Exploration ("I do and I understand")
- Practice and real-world application
- And just to have fun with math!

Getting students to get their hands "dirty" as they use mathematical thinking will build a strong foundation that will help students to truly understand a concept and be more likely to be able to apply it in the future. Let's try a few hands-on classroom activities that use puzzles to engage students in learning some important algebra and geometry concepts. On the next pages you will find a summary of the three puzzle activities and a copy of some classroom handouts for the activities. For a more detailed explanation of how to use the puzzles in your classroom, you can read the articles from *The Oregon Math Teacher (TOMT)* journals available at www.knightmath.com/resources .

The Five Square Puzzle



A good puzzle can develop problem solving and multiple content strands and be fun at the same time. This is definitely true about the five square puzzle. This is a good puzzle to use with students in an algebra or geometry class who are learning to simplify radicals. When presented correctly to students, they should see why it is useful to simplify radicals. It is best to do this activity after students have learned how to simplify radicals.

Activity Procedure Outline:

1. Object of the Puzzle: Use the pieces from the 5 squares to make one large square.
2. After students have cut out the cardstock version of the puzzle (or give them wooden puzzles) let them try the puzzle. Most students will come up with a rectangle answer. Congratulate them and then challenge them to find the square.
3. When students begin to get frustrated with a trial-and-error approach, direct them to the math on the handout.

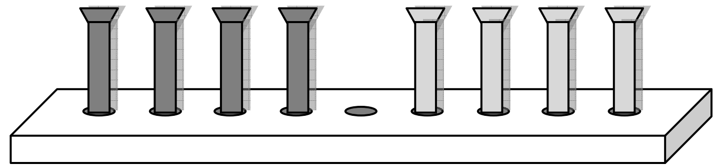
   5-square puzzle Article

   Consider the areas of the squares we let the small squares be a 2×2 square. The key is that *the area of the whole=the area of the parts*. So, the area of the large square needs to be 20 $un.^2$. So, what does the length of the side need to be? The side is $\sqrt{20}$ un.

   How do you get a length of $\sqrt{20}$ from the sides of the puzzle pieces? Well, if we simplify $\sqrt{20} = 2\sqrt{5}$, we see that we don't need a piece that is $\sqrt{20}$ units, we need something that is $\sqrt{5}$ un. Using the Pythagorean theorem on the right triangle piece, we find that the hypotenuse of the triangle is $\sqrt{5}$ units long. That's the key!
4. Now we know that we need to build a large square that has two lengths of $\sqrt{5}$ on each side. This means that the "slanted sides" must face out.

While students are solving the puzzle and using the math, they should start to get a nice concrete grasp on what it means for something to be $\sqrt{5}$ or $2\sqrt{5}$ units long. This can be a key to helping students move into the world of irrational numbers when they realize that $\sqrt{5}$ is just an exact amount even though we can't show it on our fingers.

The Leap Frog Puzzle



Recognizing numerical patterns and describing them with algebra is an important skill. The *Leap Frog Puzzle* is a good activity to teach students to use inductive reasoning to discover how many moves it takes to solve this fun and challenging puzzle.

Activity Procedure Outline:

1. Objective: Move the pegs until the two different colored groups of pegs switch sides. You may slide one peg to an open hole next to it or you may jump one peg.
2. Allow students to play with the puzzle, and most students will solve the puzzle within about 5 minutes.
3. Now give the students a new goal: Complete the puzzle in the minimum number of moves. Students can count the number of moves and you can keep track of it on the board. It should soon be evident that this is not an easy question to answer by trial and error. This opens the door for some good discussion.
    - How do we know if a method will give us the minimum number of moves? If we do not move pegs "backwards" toward the initial side, then we will have found the best solution with the minimum moves.
    - Now students need to try to complete the puzzle without moving backwards. Let them try this for a few minutes and we see that it is difficult to do.
4. Now we can use the *Wishful Thinking Method* to find a pattern that will help to answer the minimum number of moves question.

Leap Frog puzzle Article

The ***Wishful Thinking Method*** (George Polya) is a technique that engages students in active learning and allows them to be construction crew of their own understanding (If they build their knowledge, they will remember and understand it better).

Step 1: Wish for an easier Problem

For the Leap Frog puzzle, wish that there were only two pegs.

Step 2: Use inductive reasoning to find a pattern that answers the original problem.

For the Leap Frog Puzzle, Keep adding pegs to find a pattern.

- ➢ Remove all the pegs except for 1 peg on each side with 1 hole in the middle. What is the minimum number of moves to solve this 1-peg puzzle?
- ➢ Remove all the pegs except for 2 pegs on each side with 1 hole in the middle. What is the minimum number of moves to solve this 2-peg puzzle?
- ➢ Remove all the pegs except for 3 pegs on each side with 1 hole in the middle. What is the minimum number of moves to solve this 3-peg puzzle?

5. Now find a pattern to figure out what the minimum number of moves is for the 4-peg puzzle. (It should end up being 24 moves.)
6. Now extend the question to 10 pegs, or n-pegs. (Giving us 120 moves for 10 pegs, and $(n+1)^2 - 1$ for n pegs.

The Five Triangle Puzzle

Level: 5th grade and up

The relationships among the side lengths of a $30° - 60° - 90°$ triangle are crucial for much of high school level math. Getting used to working with the ratios of these side lengths can be challenging for students. This puzzle provides an opportunity for students to work with the this special right triangle in a fun way.

In this puzzle, all five pieces are to be used to form one large triangle. There are three large $30° - 60° - 90°$ triangles and two smaller ones. Note that the hypotenuse of the smaller triangle is equal in measure to the longer leg of the larger triangle.

This puzzle can be used as early as 5th grade when students begin working with angles. If the puzzle is used with students who have not studied $30° - 60° - 90°$ triangles, just use the first page of the handout and let the kids solve the triangle with some basic intuition. If students have studied $30° - 60° - 90°$ triangles, then you can really dig into the math involved and prove that there can only be two possible types of triangles.

Activity Procedure Outline:

This procedure is laid out in the student handout at the end of this packet. Here are a few notes to consider along the way:

5-triangle puzzle Article

1. Begin by letting students try to solve the puzzle with trial-and-error. This motivates the need to use our mathematical understanding. After experimenting for about 5 minutes, students will begin to ask,
   What type of triangle do we need to make?
2. At this point, you can commend the students on asking one of the key questions related to this puzzle. This is the time to begin the handout.
3. **Phase 1:** These questions will make students think about what angles we can have in a triangle. Is it possible to get a 45° angle in our final triangle? Clearly it is not.

   Question 2 should lead to the discovery that the only possible triangles are
   $$30° - 60° - 90°, \qquad 60° - 60° - 60°, \qquad or\ 30° - 150° - 30°.$$
4. **Phase 2:** This phase is for students who have learned about the ratio of side lengths in a $30° - 60° - 90°$ triangle.
   a. Question 4 should lead to areas of
   $$A(small\ triangle) = \frac{3}{2}\sqrt{3}, \qquad A(large\ triangle) = 2\sqrt{3}$$
   b. In question 5 we find that
   $$A(final\ triangle) = 2\left(\frac{3}{2}\sqrt{3}\right) + 3\left(2\sqrt{3}\right) = 9\sqrt{3}$$
   c. In questions 6-8 we can use the area to find out that which triangles are even possible and we realize that we don't have to waste time looking for a large $30° - 60° - 90°$.

5-Square Puzzle

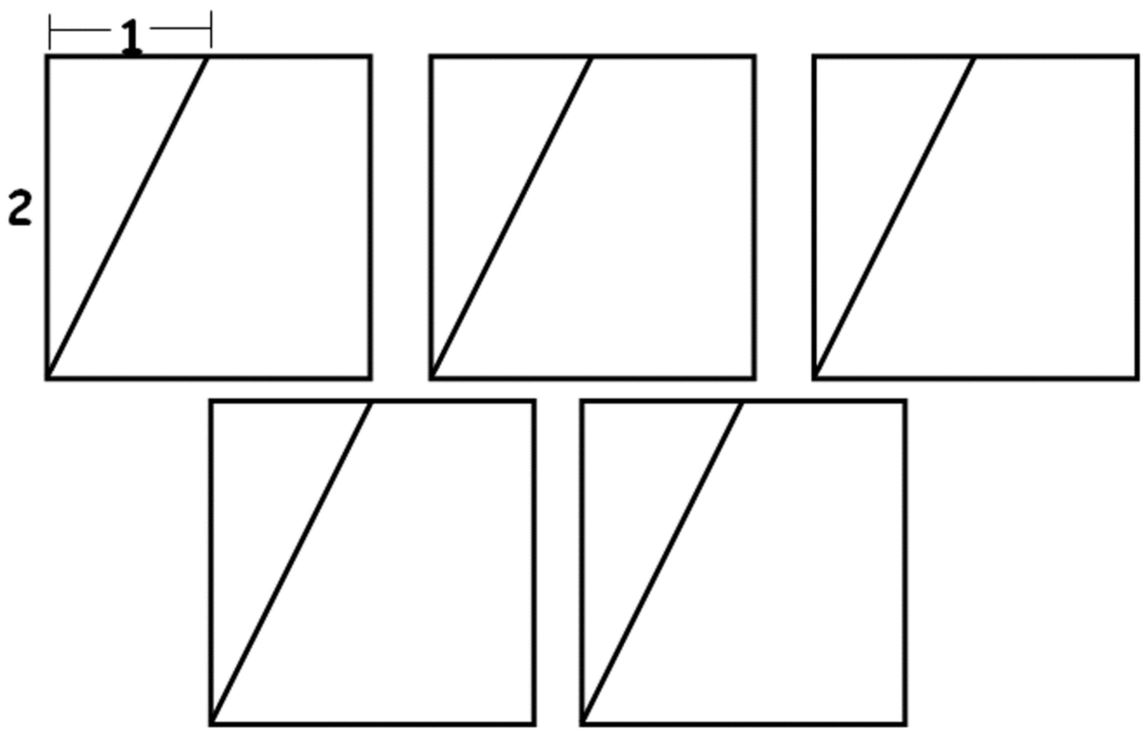
Object: Make a large square out of the pieces of the five small squares

Begin by cutting out the pieces below and move them to try to make one large square

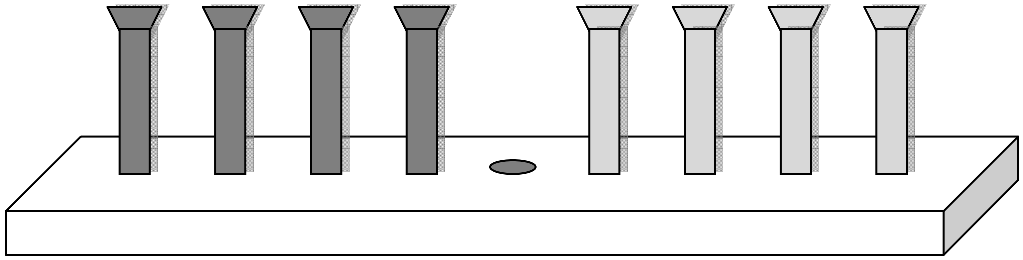
Answer the following questions to help you solve the puzzle;

1. What is the area of the small square?
2. What is the total area of the five squares?
3. What will the area of the final, large square be?
4. What will the length of one side of the final square be?
   Can this be made by putting 1's and 2's together?
5. What is the length of the hypotenuse of the triangle cut from the small square?

Now try using this new information to better attack the puzzle solution.



Leap Frog Puzzle



Object: Move the pegs one at a time so the pegs end up on the opposite sides of the board. A peg may jump over only one other peg per move or slide to an open hole next to it.

Level 1 Game: Complete the puzzle with "backwards" moves allowed.
Level 2 Game: Complete the puzzle without moving "backwards"

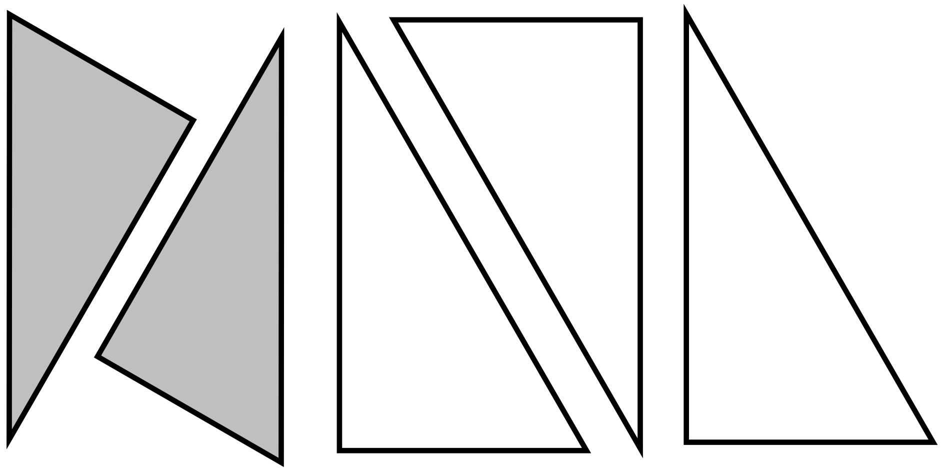
The big question: What is the minimum number of moves that it will take to swap the pegs of a ten-peg puzzle?

Fill in the chart to find a pattern that will help you answer the big question.

Number of Pegs	Number of Moves
1	
2	
3	
4	
.	
.	
.	
10	

Five Triangle Puzzle

The object of this puzzle is to use all five right triangles below to form one large triangle.



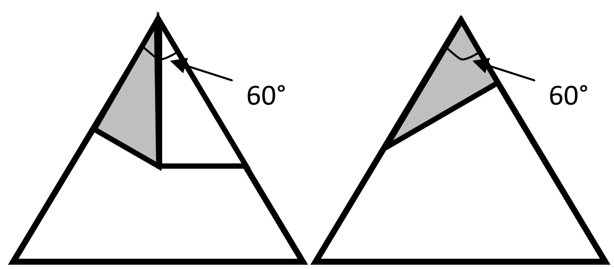
Begin by cutting out the triangle above. Notice that there are two smaller right triangles and three larger right triangles, and the hypotenuse of the smaller triangle is equal in measure to the longer leg of the larger triangle.

Questions to consider

There is one more twist to this puzzle… it has more than one solution! The answers to the following questions will help you to find all the solutions to the 5-triangle puzzle.

It is important to note that the puzzle piece are not just your everyday right triangles, these are *special right triangles.* They are special because their angle measures are 30°, 60°, and 90°, making it half of an equilateral triangle. We first need to think about the possible types of triangles that we can end up with in our final triangle.

Phase 1: Consider possible triangles



1. When we use the pieces to build the final triangle, one possible angle in the final triangle could be 60° by either placing two 30° angles in a corner or placing one 60° in the corner as shown in the example at the right.

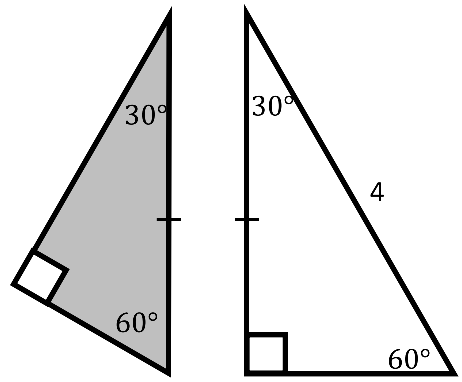
   List all the possible acute angles (no greater than180°) that could be made by combining one or more of the puzzle piece angles (30°, 60°, or 90°). There are exactly 5 such possible angles.

   __________ __________ __________ __________ __________

2. We know that the angles in a triangle must have a sum of 180°. Using the possible final angles you found in #1 above, what are all the possible combinations of angle measures that we could end up in our final triangle?

Phase 2: Digging deeper with radicals and areas

Leave all lengths and area measures in simplest radical form



3. If the hypotenuse of the large triangle is 4 units, find the length of each side of the two triangles and label them.

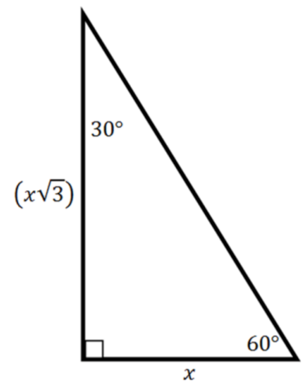
4. Find the area of the two triangles in simplest radical form:

   A(Small Triangle)=

   A(Large Triangle)=

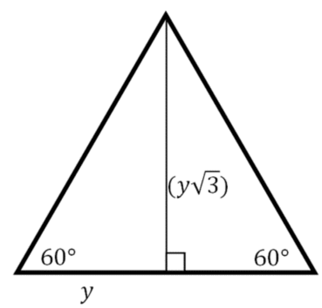
5. When we combine all five triangles, what will the area of the complete triangle be? (*Hint: The whole is the sum of its parts.*)

   A(Final Triangle)=



6. Let's find out if it is possible to make one large 30°-60°-90° triangle with the puzzle pieces. Use the area formula for a triangle and your answer from #5 to figure out what *x* would have to be in the triangle to the right.

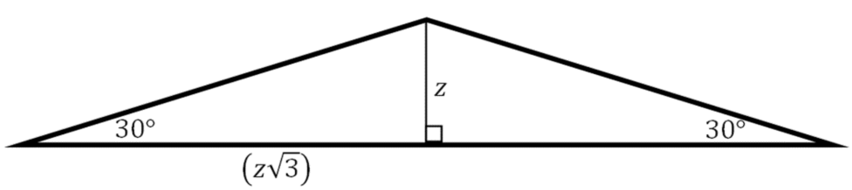
   Is it possible to use the puzzle pieces to make a length of this *x*?
   So, is it possible to have a final triangle with angles of 30°-60°-90°?



7. Now, let's find out if it is possible to make one large 60°-60°-60° triangle with the puzzle pieces. Use the area formula for a triangle and your answer from #5 to figure out what *y* would have to be in the triangle to the right.

   What would the length of the entire base be?
   Is it possible to use the puzzle pieces to make this base length?
   So, is it possible to have a final triangle with angles of 60°-60°-60°?

8. Finally, is it possible to make one large $30° - 150° - 30°$ triangle?
   If so, what would *z* have to be below? How long is the base?



Conclusion: After answering these questions, you should know what the two possible types of final triangles are. Now, use this information to help you find both solutions to the puzzle.

Five Triangle Puzzle Pieces

Cut out the puzzle pieces below and use them to make one large triangle. Is there more than one solution?