5.1-5.2: An introduction to Probability

5.1: Probability Rules

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BUREAU OF LABOR STATISTICS
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"The unemployment rate declined by 0.2 percentage point to 3.6 percent in March, and the number of unemployed persons decreased by 318,000 to 6.0 million. These measures are little different from their values in February 2020 (3.5 percent and 5.7 million, respectively), prior to the coronavirus (COVID19) pandemic"

"Among the major worker groups, the unemployment rate for adult women (3.3 percent) declined in March. The jobless rates for adult men (3.4 percent), teenagers (10.0 percent), Whites (3.2 percent), Blacks (6.2 percent), Asians (2.8 percent), and Hispanics (4.2 percent) showed little change over the month."

Questions we might consider:

- What do these numbers tell us?
- How likely are these statistics to happen?
- Is this "little difference" statistically significant

These types of questions lead us to think about statistics with a probability perspective.

Probability Rules

The theory of probability was developed by Pierre de Fermat and Blaise Pascal in the 1600's as they wrote letters to each other considering the chances of winning a game.

Probability is a measure of the likelihood of a random phenomenon or chance behavior. Probability describes the long-term proportion with which a certain **outcome/result** will occur in situations with short-term uncertainty.

Probability deals with experiments that yield random short-term results or **outcomes**, yet reveal long-term predictability. **The long-term proportion in which a certain outcome is observed is the probability of that outcome/result.**

The Law of Large Numbers
As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome.

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Approximating Probabilities Using the Empirical Approach

The probability of an event/result, E, is approximately the number of times event E is observed divided by the number of repetitions of the experiment.

$$P(E) \approx \text{relative frequency of } E$$

$$P(E) = \frac{Frequency\ of\ E}{Number\ of\ trials\ of\ an\ experiment}$$

In probability, an **experiment** is any process that can be repeated in which the results are uncertain.

Let's Experiment

Experiment 1: Flipping a coin,

- What is the probability of getting heads?

- What is the probability of getting tails?

Experiment 2: Flipping a Hershey's Kiss,

- What is the probability of landing point up?

- What is the probability of landing point down?

Rules of probabilities

1. The probability of any event /result must be greater than or equal to 0 and less than or equal to 1.
   That is, $0 \leq P(result) \leq 1$.
2. The sum of the probabilities of all outcomes/results must equal 1.

Probability Model

A **probability model** lists the possible outcomes/results of a probability experiment and each outcome's probability. A probability model must satisfy rules 1 and 2 of the rules of probabilities.

EXAMPLE A Probability Model

In a bag of peanut M&M milk chocolate candies, the colors of the candies can be brown, yellow, red, blue, orange, or green. Suppose that a candy is randomly selected from a bag. The table shows each color and the probability of drawing that color. Verify this is a probability model.

Color	Probability
Brown	0.12
Yellow	0.15
Red	0.12
Blue	0.23
Orange	0.23
Green	0.15

- All probabilities are between 0 and 1, inclusive.
- Because $0.12 + 0.15 + 0.12 + 0.23 + 0.23 + 0.15 = 1$, rule 2 (the sum of all probabilities must equal 1) is satisfied.

EXAMPLE Building a Probability Model

Pass the Pigs™ is a Milton-Bradley game in which pigs are used as dice. Points are earned based on the way the pig lands. There are six possible outcomes when one pig is tossed. A class of 52 students rolled pigs 3,939 times. The number of times each outcome occurred is recorded in the table at right

Outcome	Frequency
Side with no dot	1344
Side with dot	1294
Razorback	767
Trotter	365
Snouter	137
Leaning Jowler	32

(a) Use the results of the experiment to build a probability model for the way the pig lands.

(b) Estimate the probability that a thrown pig lands on the "side with dot".

(c) Would it be unusual to throw a "Leaning Jowler"?

The Classical Method

The classical method of computing probabilities requires *equally likely outcomes.*
An experiment is said to have **equally likely outcomes** when each simple event/result has the same probability of occurring.

Computing Probability Using the Classical Method

If an experiment has n equally likely outcomes and if the number of ways that an event E can occur is m, then the probability of E, $P(E)$ is

$$P(E) = \frac{number\ of\ ways\ E\ can\ occur}{numbe\ of\ possible\ outcomes} = \frac{m}{n}$$

Example

Suppose a "fun size" bag of M&Ms contains: **9 brown, 6 yellow, 7 red, 4 orange, 2 blue, and 2 green.** Suppose that a candy is randomly selected.

(a) What is the probability that it is yellow?
(b) What is the probability that it is blue?
(c) Comment on the likelihood of the candy being yellow versus blue.

5.2: The Addition Rule and Complements

Consider this:
Suppose you are playing a card game with these cards in the picture and you are going to pick one card at random. Find these probabilities

a) $P(heart) =$

b) $P(diamond) =$

c) $P(ace) =$

d) $P(heart\ or\ diamond) =$

e) $P(ace\ or\ heart) =$

Two events are **disjoint** if they have no outcomes in common. Another name for disjoint events is **mutually exclusive** events.

Addition Rule for Disjoint Events
If E and F are disjoint (or mutually exclusive) events, then

$$P(E\ or\ F) = P(E) + P(F)$$

In general, if E, F, G, . . . each have no outcomes in common (they are pairwise disjoint), then

$$P(E \text{ or } F \text{ or } G \text{ or } \ldots) = P(E) + P(F) + P(G) \ldots$$

Example

The probability model to the right shows the distribution of the number of rooms in housing units in the United States.

Number of Rooms in Housing Unit	**Probability**
One	0.010
Two	0.032
Three	0.093
Four	0.176
Five	0.219
Six	0.189
Seven	0.122
Eight	0.079
Nine or more	0.080

(a) Verify that this is a probability model.

All probabilities are between 0 and 1, inclusive.

0.010 + 0.032 + … + 0.080 = 1

(b) What is the probability a randomly selected housing unit has two or three rooms?

(c) What is the probability a randomly selected housing unit has one or two or three rooms?

Complement of an Event

Let *E* denote an event. The complement of *E*, denoted not E, are not outcomes in the event *E*.

Example:

According to the American Veterinary Medical Association, 31.6% of American households own a dog. What is the probability that a randomly selected household does not own a dog?

5.3: Independence and the Multiplication Rule

Dependent vs. Independent

Two events E and F are **independent** if the occurrence of event E in a probability experiment does not affect the probability of event F. Two events are **dependent** if the occurrence of event E in a probability experiment affects the probability of event F.

Consider These:

Determine if these are independent of dependent.

(a) Suppose you draw a card from a standard 52-card deck of cards and then roll a die. Are the events "draw a heart" and "roll an even number" independent?

(b) Suppose two 40-year old women who live in the United States are randomly selected. Are the events "woman 1 survives the year" and "woman 2 survives the year" independent.

(c) Suppose two players from a high school football team are chosen to be captains at random. Are the events "player 1 is a senior" and "player 2 is a senior" independent?

Multiplication Rule for Independent Events

If E and F are independent events, then

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

Example

The probability that a randomly selected female aged 60 years old will survive the year is 99.186% according to the National Vital Statistics Report, Vol. 47, No. 28. What is the probability that two randomly selected 60 year old females will survive the year?

Multiplication Rule for *n* Independent Events

If n events are independent, find the probability of all of them happening together by **multiplying** the probabilities of them happening by themselves.

What is the probability that four randomly selected 60 year old females will survive the year?