6.2: The Binomial Probability Distribution

What is a Binomial Probability Experiment?

Criteria for a Binomial Probability Experiment
An experiment is said to be a **binomial experiment** if:

1. The experiment is performed a **fixed number of times**.
   Each repetition of the experiment is called a **trial**.
2. The trials are **independent**.
   This means the outcome of one trial will not affect the outcome of the other trials.
3. For each trial, there are **two mutually exclusive (or disjoint) outcomes**, success or failure.
4. The **probability of success is fixed** for each trial of the experiment.

Notation Used in the Binomial Probability Distribution

- There are n independent trials of the experiment.
- Let p denote the probability of success so that $1 - p$ is the probability of failure.
- Let X be a binomial random variable that denotes the number of successes in n independent trials of the experiment.
  So, $0 \leq x \leq n$.

Example ***Which of the following are binomial experiments?***

(a) A player rolls a pair of fair die 10 times. The number X of 7's rolled is recorded.

(b) The 11 largest airlines had an on-time percentage of 84.7% in November, 2001 according to the Air Travel Consumer Report. In order to assess reasons for delays, an official with the FAA randomly selects flights until she finds 10 that were not on time. The number of flights X that need to be selected is recorded.

(c) In a class of 30 students, 55% are female. The instructor randomly selects 4 students. The number X of females selected is recorded.

Compute Probabilities of Binomial Experiments

The **multiplication rule** of probabilities is the key to finding the probability of a certain outcome happening.

> **Multiplication Rule for Independent Events**
> If E and F are independent events, then
> $$P(E \text{ and } F) = P(E) \cdot P(F)$$

> **Multiplication Rule for n Independent Events**
> If n events are independent, find the probability of all of them happening together by **multiplying** the probabilities of them happening by themselves.

Example – Air Travel:
According to the Air Travel Consumer Report, the 11 largest air carriers had an on-time percentage of 79.0% in May, 2008. Suppose that 3 flights are randomly selected from May, 2008 and the number of on-time flights X is recorded.
Construct a probability distribution for the random variable X using a tree diagram.
Let O represent "On Time", and L represent "Late".

1st Flight	2nd Flight	3rd Flight	Outcomes	Probabilities

What is the probability that **exactly one flight** will be late?

Note: On Calculator use 2nd , [VARS] , binompdf (n, p, x) = P(x)

> **Binomial Probability Distribution Function**
> The probability of obtaining x successes in n independent trials of a binomial experiment is given by
> $$P(x) = {}_nC_x p^x (1-p)^{n-x} \quad x = 0,1,2,\ldots,n$$
> where p is the probability of success.

Building a Probability Distribution using TI84 binompdf
Use the **binompdf(n,p,x)** function and the on-time percentage of 79.0% in the last problem to find the probabilities of the given number of on-time planes out of 3 randomly selected planes.

Number of On-Time Planes	Probability
0	$P(0) =$
1	$P(1) =$
2	$P(2) =$
3	$P(3) =$

<u>Consider these</u> using the table you just made:

a) What is the probability that *no more than* two planes will be on time?
That is, find $P(x \leq 2)$

b) What is the probability that *at least* two planes will be on time?
That is, find $P(x \geq 2)$

Cumulative Distribution Function

The **cumulative distribution function** (*binomcdf* on your TI84) will give you the sum of all the probabilities less than or equal to a given success rate.

<u>Try these</u> using [2nd] – [Vars] – "binomcdf(n,p,x)":

a) What is the probability that *no more than* two planes will be on time?
That is, find $P(x \leq 2)$

$$P(x \leq 2) = binomcdf(3, .79, 2) =$$

b) What is the probability that *at least* two planes will be on time?
That is, find $P(x \geq 2)$

$$P(x \geq 2) = 1 - P(x \leq 1)$$
$$= 1 - binomcdf(3, .79, 1) =$$

Keep in mind these phrases:

Phrase	Math Symbol
"at least" or "no less than" or "greater than or equal to"	$\geq$
"more than" or "greater than"	$>$
"fewer than" or "less than"	$<$
"no more than" or "at most" or "less than or equal to	$\leq$
"exactly" or "equals" or "is"	$=$

<u>Example: Experian Automotive</u>

According to the Experian Automotive, 35% of all car-owning households have three or more cars.
Note: Write it out, then use the calculator.

(a) In a random sample of 20 car-owning households, what is the probability that **exactly 5** have three or more cars?

$P(5) =$

Calculator: 2nd VARS binompdf (20, .35, 5) = P(5) = .1272

(b) In a random sample of 20 car-owning households, what is the probability that **less than 4** have three or more cars?

$P(X < 4) = P(X \leq 3) =$

Calculator: 2nd VARS binompdf (20, .35, 5) = P(5) = .1272

(c) In a random sample of 20 car-owning households, what is the probability that **at least 4** have three or more cars?

$P(X \geq 4) = 1 - P(X \leq 3) =$

Compute the Mean and Stand. Dev. Of a Binomial Random Variable

Mean (or Expected Value) and Standard Deviation of a Binomial Random Variable

A binomial experiment with n independent trials and probability of success p has a mean and standard deviation given by the formulas

$$\mu_X = np \quad \text{and} \quad \sigma_X = \sqrt{np(1-p)}$$

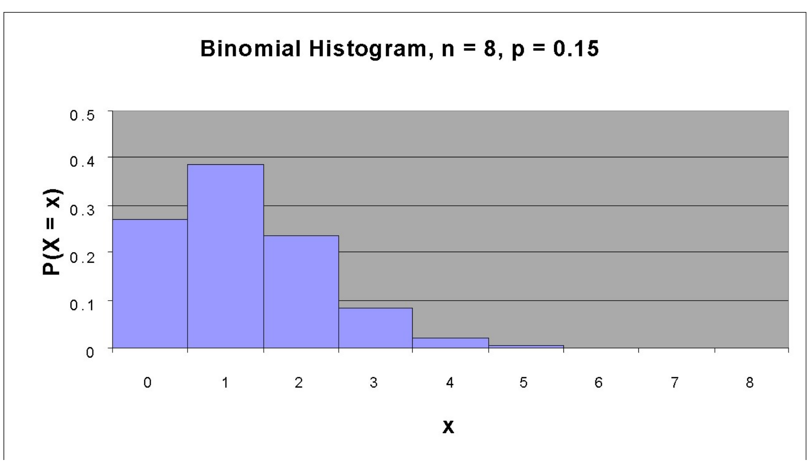
Example – More Cars

According to the Experian Automotive, 35% of all car-owning households have three or more cars. In a simple random sample of 400 car-owning households, determine the mean and standard deviation number of car-owning households that will have three or more cars.

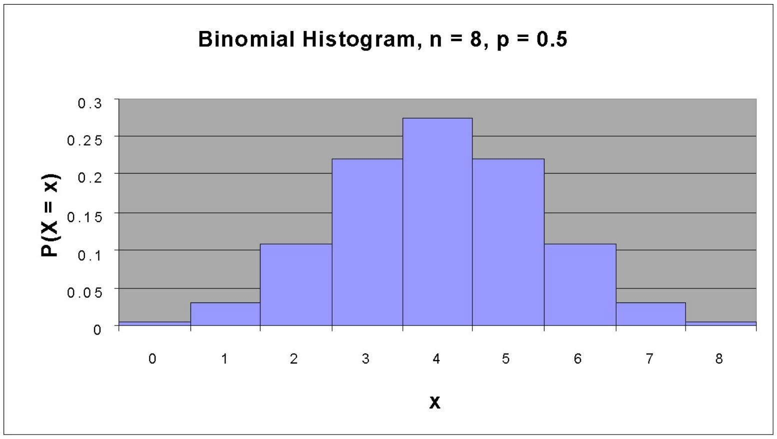
Binomial Probability Histograms

Example: Construct a binomial probability histogram for the parameters given. Comment on the shape of the distribution.

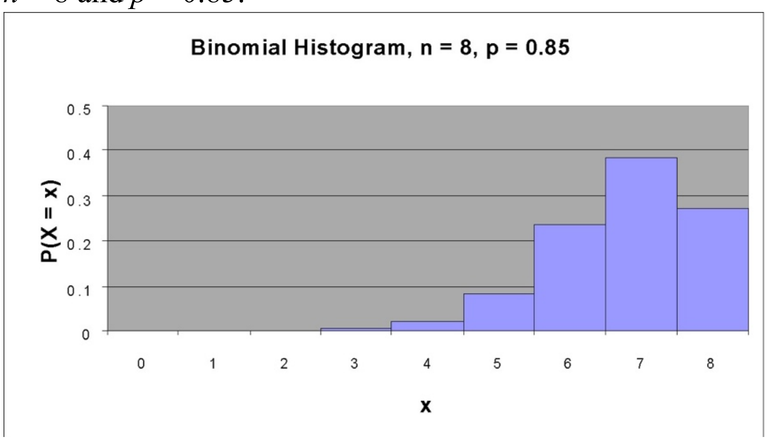
(a) $n = 8$ and $p = 0.15$.



(b) $n = 8$ and $p = 0.5$.



(c) $n = 8$ and $p = 0.85$.



Example: Make your own

Construct a binomial probability histogram with $n = 25$ and $p = 0.8$. Comment on the shape of the distribution.

For a fixed probability of success, p, as the number of trials n in a binomial experiment increase, the probability distribution of the random variable X becomes bell-shaped.

As a general rule of thumb, if $np(1-p) \geq 10$, then the probability distribution will be approximately bell-shaped.

Use the Empirical Rule to identify unusual observations in a binomial experiment.

- The Empirical Rule states that in a bell-shaped distribution about 95% of all observations lie within two standard deviations of the mean.
- 95% of the observations lie between $\mu - 2\sigma$ and $\mu + 2\sigma$.
- Any observation that lies outside this interval may be considered unusual because the observation occurs less than 5% of the time.

Example: Unusual

According to the Experian Automotive, 35% of all car-owning households have three or more cars. A researcher believes this percentage is higher than the percentage reported by Experian Automotive. He conducts a simple random sample of 400 car-owning households and found that 162 had three or more cars. Is this result unusual ?