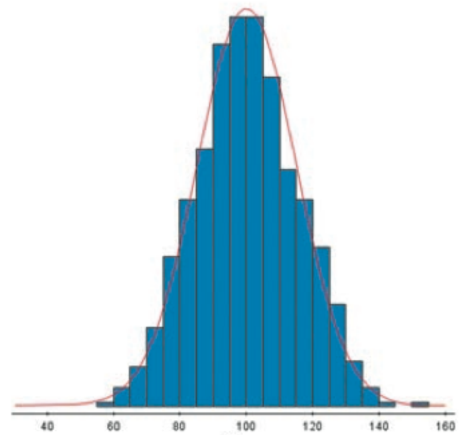
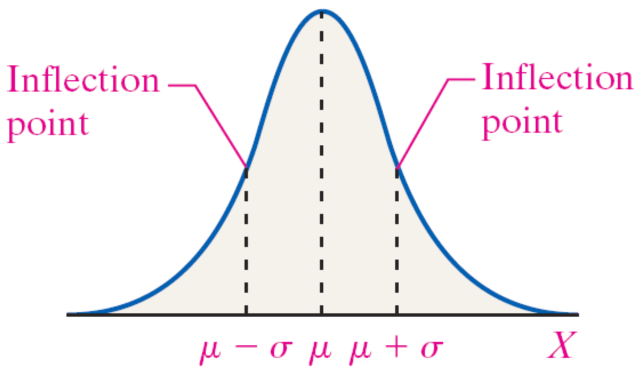
7.1 The Graph of a Normal Curve

Relative frequency histograms that are symmetric and bell-shaped are said to have the shape of a **normal curve.**

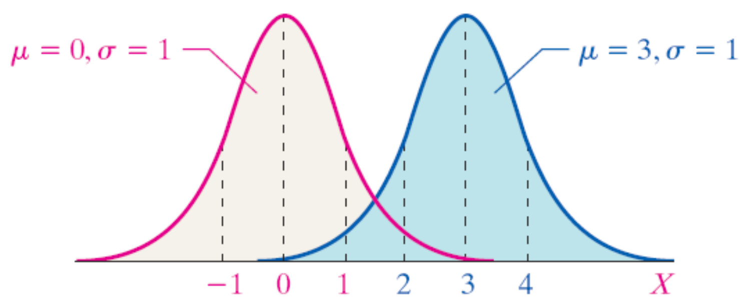




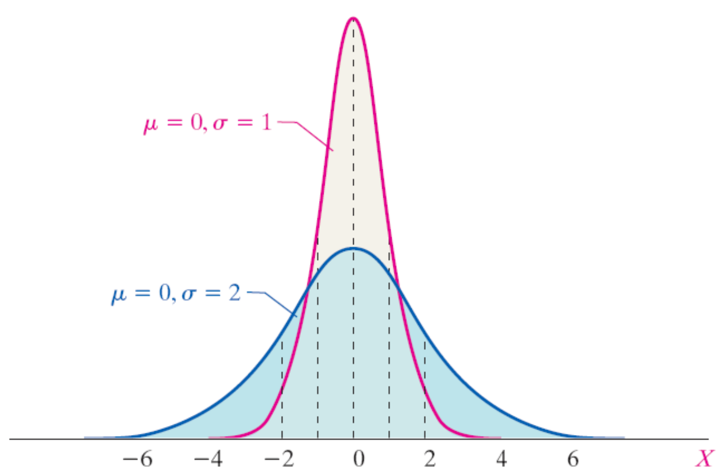
If a continuous random variable is **normally distributed**, or has a **normal probability distribution**, then a relative frequency histogram of the random variable has the shape of a normal curve (bell-shaped and symmetric).

<u>Some Examples:</u>

Here are two normal curves that have the same σ but different μ.
How are they the same, and how are they different?



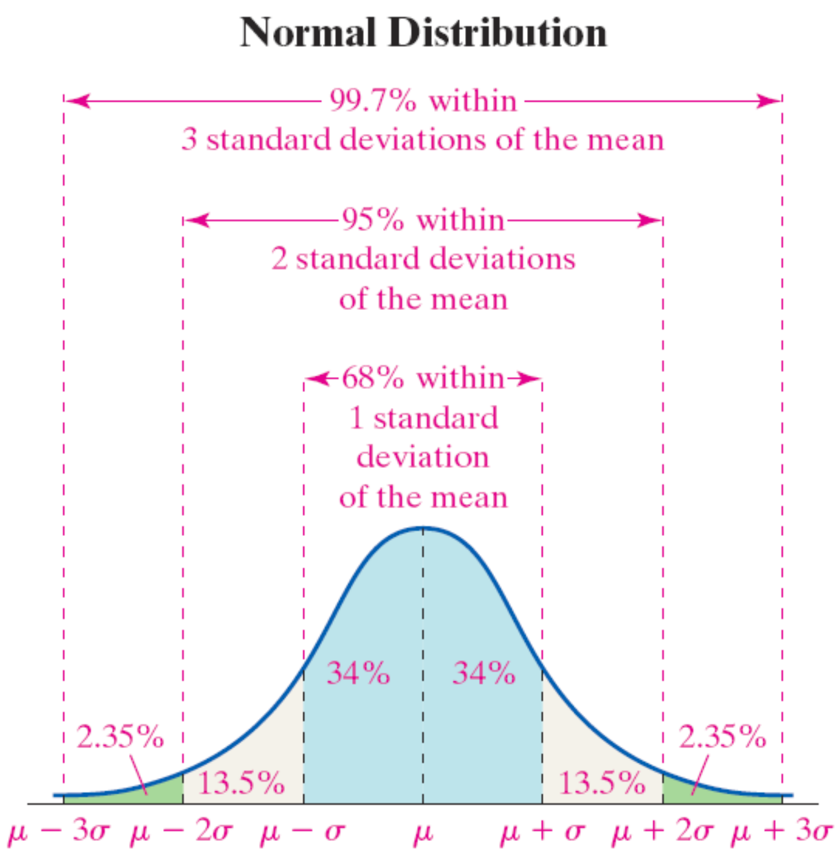
Here are two normal curves that have the same μ but different σ.
How are they the same, and how are they different?



Properties of the Normal Curve

Properties of the Normal Density Curve

1. It is s**ymmetric about its mean, *μ*.**
2. Because mean = median = mode, the curve has a single peak and the highest point occurs at *x* = *μ*.
3. **It has inflection points at *μ* – *σ* and *μ* – *σ***
4. The a**rea under the curve is 1**.
5. The area under the curve **to the right of *μ* equals the area under the curve to the left of *μ***, which equals 1/2.
6. As x increases without bound (gets larger and larger), the graph approaches, but never reaches, the horizontal axis. As x decreases without bound (gets more and more negative), the graph approaches, but never reaches, the horizontal axis.
7. **The Empirical Rule:**
   - Approx. 68% of the area under the normal curve is between x = μ – σ and x = μ + σ;
   - approximately 95% of the area is between x = μ – 2σ and x = μ + 2σ
   - approximately 99.7% of the area is between x = μ – 3σ and x = μ + 3σ.



Area and the Normal Density Function

Example

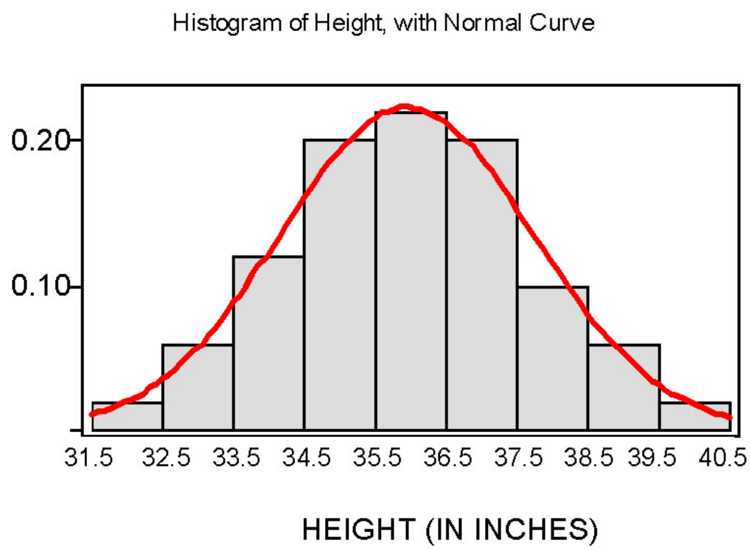
The data on the next slide represent the heights (in inches) of a random sample of 50 two-year old males.
(a) Draw a histogram of the data using a lower class limit of the first class equal to 31.5 and a class width of 1.
(b) Do you think that the variable "height of 2-year old males" is normally distributed?

36.0	36.2	34.8	36.0	34.6	38.4	35.4	36.8
34.7	33.4	37.4	38.2	31.5	37.7	36.9	34.0
34.4	35.7	37.9	39.3	34.0	36.9	35.1	37.0
33.2	36.1	35.2	35.6	33.0	36.8	33.5	35.0
35.1	35.2	34.4	36.7	36.0	36.0	35.7	35.7
38.3	33.6	39.8	37.0	37.2	34.8	35.7	38.9

37.2 39.3

(c) At the right, we have a normal density curve drawn over the histogram.

How does the area of the rectangle corresponding to a height between 34.5 and 35.5 inches relate to the area under the curve between these two heights



Area under a Normal Curve

Suppose that a random variable X is normally distributed with mean μ and standard deviation σ.
The area under the normal curve for any interval of values of the random variable X represents either

- **the proportion** of the population **with the characteristic described** by the interval of values or
- **the probability** that a randomly selected individual from the population **will have the characteristic** described by the interval of values.

Example:

The weights of giraffes are approximately normally distributed with mean $\mu = 2200$ pounds and standard deviation $\sigma = 200$ pounds.

(a) Draw a normal curve with the parameters labeled.

(b) Shade the area under the normal curve to the left of $x = 2100$ pounds.

(c) Suppose that the area under the normal curve to the left of $x = 2100$ pounds is 0.3085. Provide two interpretations of this result.