10.1: The Language of Hypotheses

Determining the Null and Alternative Hypotheses

A **hypothesis** is a statement regarding a characteristic of one or more populations.

Example: Claims regarding a characteristic of a single population

- In 2008, 62% of American adults regularly volunteered their time for charity work. A researcher believes that this percentage is different today.

- According to a study published in March, 2006 the mean length of a phone call on a cellular telephone was 3.25 minutes. A researcher believes that the mean length of a call has increased since then.

- Using an old manufacturing process, the standard deviation of the amount of wine put in a bottle was 0.23 ounces. With new equipment, the quality control manager believes the standard deviation has decreased.

CAUTION!

We test these types of statements using sample data because it is usually impossible or impractical to gain access to the entire population.
If population data are available, there is no need for inferential statistics.

Hypothesis testing is a procedure, based on sample evidence and probability, used to test statements regarding a characteristic of one or more populations.

Steps of Hypothesis Testing

1. Make a statement regarding the nature of the population.
2. Collect evidence (sample data) to test the statement.
3. Analyze the data to assess the plausibility of the statement.

Two Hypotheses

- The **null hypothesis**, denoted H_0, is a statement to be tested. The null hypothesis is a statement of no change, no effect or no difference and is assumed true until evidence indicates otherwise.
- The **alternative hypothesis**, denoted H_1, is a statement that we are trying to find evidence to support.

Three ways to set up the null and alternative hypotheses:

1. Equal versus not equal hypothesis (**two-tailed test**)
   H_0: parameter = some value
   H_1: parameter ≠ some value
2. Equal versus less than (**left-tailed test**)
   H_0: parameter = some value
   H_1: parameter < some value
3. Equal versus greater than (**right-tailed test**)
   H_0: parameter = some value
   H_1: parameter > some value

In Other Words…

The null hypothesis is a statement of *status quo* or *no difference* and always contains a statement of equality. The null hypothesis is assumed to be true until we have evidence to the contrary. We seek evidence that supports the statement in the alternative hypothesis.

Example: Forming Hypotheses

For each of the following claims, determine the null and alternative hypotheses. State whether the test is two-tailed, left-tailed or right-tailed.

a) In 2008, 62% of American adults regularly volunteered their time for charity work. A researcher believes that this percentage is different today.

   Null Hypothesis:

   Alternative Hypothesis:

   Type of Test:

b) According to a study published in March, 2006 the mean length of a phone call on a cellular telephone was 3.25 minutes. A researcher believes that the mean length of a call has increased since then.

   Null Hypothesis:

   Alternative Hypothesis:

   Type of Test:

c) Using an old manufacturing process, the standard deviation of the amount of wine put in a bottle was 0.23 ounces. With new equipment, the quality control manager believes the standard deviation has decreased.

Null Hypothesis:

Alternative Hypothesis:

Type of Test:

Type I and Type II Errors

We have two hypotheses. The key is that we are deciding to either

a) Reject the Null Hypothesis, or
b) Accept the Null Hypotheis

Four Outcomes of Hypothesis Testing

1. Reject the null hypothesis when the alternative hypothesis is true. This decision would be correct
2. Do not reject the null hypothesis when the null hypothesis is true. This decision would be correct.
3. Reject the null hypothesis when the null hypothesis is true. This decision would be incorrect. This type of error is called a **Type I error.**
4. Do not reject the null hypothesis when the alternative hypothesis is true. This decision would be incorrect. This type of error is called a **Type II error.**

Example:Type I and II Errors

For each of the following claims, explain what it would mean to make a Type I error. What would it mean to make a Type II error?

a) In 2008, 62% of American adults regularly volunteered their time for charity work. A researcher believes that this percentage is different today.

*A **Type I** error would occur if the sample evidence leads the researcher to conclude…*

*A **Type II** error would occur if the sample evidence leads the researcher to conclude …*

b) According to a study published in March, 2006 the mean length of a phone call on a cellular telephone was 3.25 minutes. A researcher believes that the mean length of a call has increased since then.

*A **Type I** error would occur if the sample evidence leads the researcher to conclude…*

*A **Type II** error would occur if the sample evidence leads the researcher to conclude …*

How do we deal with these possible errors:

Error Probabilities:

α = P(Type I Error)
= P(rejecting H_0 when H_0 is true)
β = P(Type II Error)
= P(not rejecting H_0 when H_1 is true)

Key Points

- The probability of making a Type I error, α, is chosen by the researcher *before* the sample data is collected.
- The **level of significance**, α, is the probability of making a Type I error.
- As the probability of a Type I error increases, the probability of a Type II error decreases, and vice-versa.

Stating Conclusions to Hypothesis Tests

CAUTION!

We never "accept" the null hypothesis, because, without having access to the entire population, we don't know the exact value of the parameter stated in the null.
Rather, we say that **we do not reject the null hypothesis**. This is just like the court system. We never declare a defendant "innocent", but rather say the defendant is "not guilty".

Example: More on Cell Phones
According to a study published in March, 2006 the mean length of a phone call on a cellular telephone was 3.25 minutes. A researcher believes that the mean length of a call has increased since then.

a) Suppose the sample evidence indicates that the null hypothesis should be rejected. State the wording of the conclusion.

"The statement in the alternative hypothesis is that…

Since the null hypothesis ($\mu = 3.25$) is rejected, there is sufficient evidence to conclude…

b) Suppose the sample evidence indicates that the null hypothesis should not be rejected. State the wording of the conclusion.

"Since the null hypothesis ($\mu = 3.25$) is ***not rejected****, there is* ***insufficient evidence*** *to conclude that…*