10.3: Hypothesis Tests for a Population Mean

Test Hypotheses about a Mean

To test hypotheses regarding the population mean assuming the population standard deviation is unknown, we use the *t*-distribution rather than the *Z*-distribution. When we replace σ with *s*,

$$\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

Using the ***t*-distribution** with $n-1$ degrees of freedom.

Testing Hypotheses Regarding a Population Mean

To test hypotheses regarding the population mean, we use the following steps, provided that:

- The sample is obtained using simple random sampling.
- The sample has no outliers, and the population from which the sample is drawn is normally distributed or the sample size is large ($n \geq 30$).
- The sampled values are independent of each other.

Steps and Example :

Assume the resting metabolic rate (RMR) of healthy males in complete silence is 5710 kJ/day. Researchers measured the RMR of 45 healthy males who were listening to calm classical music and found their mean RMR to be 5708.07 with a standard deviation of 992.05.

At the $\alpha = 0.05$ level of significance, is there evidence to conclude that the mean RMR of males listening to calm classical music is different than 5710 kJ/day?

Step 1: Determine the null and alternative hypotheses. Again, the hypotheses can be structured in one of three ways:

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu < \mu_0$	$H_1: \mu > \mu_0$

Note: μ_0 is the assumed value of the population mean.

Step 2: Select a level of significance, α, based on the seriousness of making a Type I error.

Step 3: Compute the **test statistic**

$$t_0 = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$$

with $n-1$ degrees of freedom.

Step 1: $H_0: \mu = 5710$; $H_1: \mu \neq 5710$

Step 2: The Level of significance is

$$\alpha = 0.05$$

Step 3:

$t_0 =$

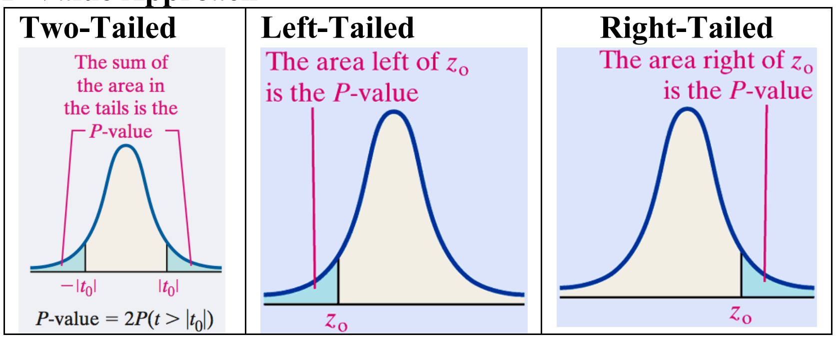
Calculator: [Stat]- [Tests]-[Ttest]

Step 4:

Classical Approach: Compare the critical value with the test statistic:

***P*-Value Approach:** Find *P*-value using a *t*-distribution table or calculator.

***P*-Value Approach**



If the *P*-value < *α*, reject the null hypothesis.

Step 5: State the conclusion

<u>**Classical Approach**</u>

Step 4: Since this is a 2-tailed approach, the critical values at

$$\alpha = 0.05$$

and

$$n - 1 = 45 - 1 = 44$$

degrees of freedom,

$t_{.975} =$

And

$t_{.025} = -t_{.975} =$

Step 5: Since the test statistic,

$$t_0 = -0.013$$

is between the critical values (i.e. it's **not outside** the critical values), then we **fail to reject the null hypothesis.**

<u>**P-Value Approach**</u>

Step 4: Use your calculator to find the P-Value **Calculator: [Stat]- [Tests]-[Ttest]**

$\boldsymbol{P} =$

Step 5: Ask, "Is this greater than the level of significance, $\alpha = 0.05$?"

It is, so we **fail to reject the null hypothesis.**

That is, the probability that this could happen with a true pop. mean of 5710 is greater than 5%. So, it's not "impossible" to get this result of $\bar{x} = 5708.07$.

Step 6: Conclusion (Is the sufficient evidence?)

There is __________________________evidence at the $\alpha = 0.05$ level of significance to conclude that the mean RMR of males listening to calm classical music differs from 5710 kJ/day.

Not: The procedure is **robust**, which means that minor departures from normality will not adversely affect the results of the test. However, for small samples, if the data have outliers, the procedure should not be used.

Example: Testing a Hypothesis about a Population Mean, Small Sample.

According to the United States Mint, quarters weigh 5.67 grams. A researcher is interested in determining whether the "state" quarters have a weight that is different from 5.67 grams. He randomly selects 18 "state" quarters, weighs them and obtains the following data.

5.70	**5.67**	**5.73**	**5.61**	**5.70**	**5.67**
5.65	**5.62**	**5.73**	**5.65**	**5.79**	**5.73**
5.77	**5.71**	**5.70**	**5.76**	**5.73**	**5.72**

At the α = 0.05 level of significance, is there evidence to conclude that state quarters have a weight different than 5.67 grams?

Solution:

**Since it's a small sample, we have to verify that it's normally distributed without outliers before proceeding. Check 1-var stats and box plot.

Step 1: H_0: H_1:

Type of test:

Step 2: The level of significance is…

Step 3: $\bar{x} =$ $s =$ $t_0 = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$

Step 4: Find t-statistic and P-value.

Step 5: Do we reject the null hypothesis?

Step 6: Conclusion: