8.2: Distribution of the Sample Proportion

Describe the Sampling Distribution of a Sample Proportion

Point Estimate of a Population Proportion

Suppose that a random sample of size n is obtained from a population in which each individual either does or does not have a certain characteristic. The sample proportion, denoted $\hat{p}$ (read "p-hat") is given by

$$\hat{p} = \frac{x}{n}$$

where x is the number of individuals in the sample with the specified characteristic.
The sample proportion $\hat{p}$ is a statistic that estimates the population proportion, p.

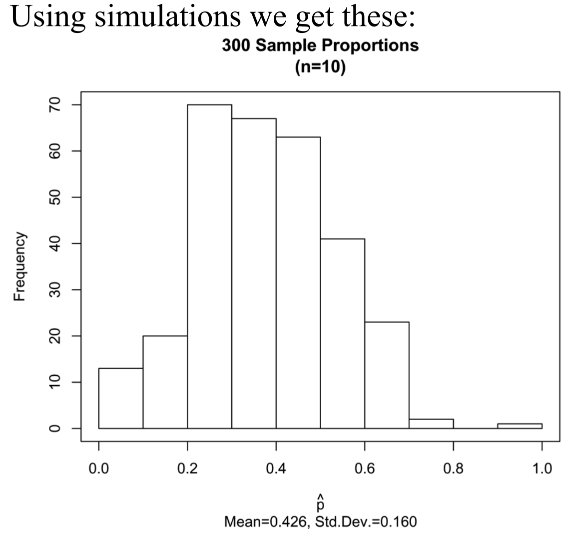
Example:

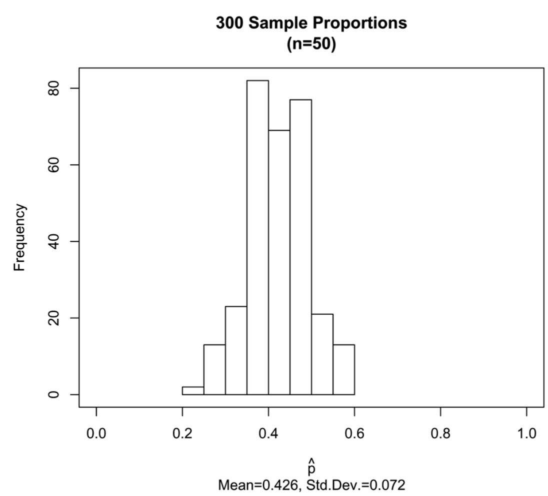
In a Quinnipiac University Poll conducted in May of 2008, 1745 registered voters nationwide were asked whether they approved of the way George W. Bush is handling the economy. 349 responded "yes". Obtain a point estimate for the proportion of registered voters who approve of the way George W. Bush is handling the economy.

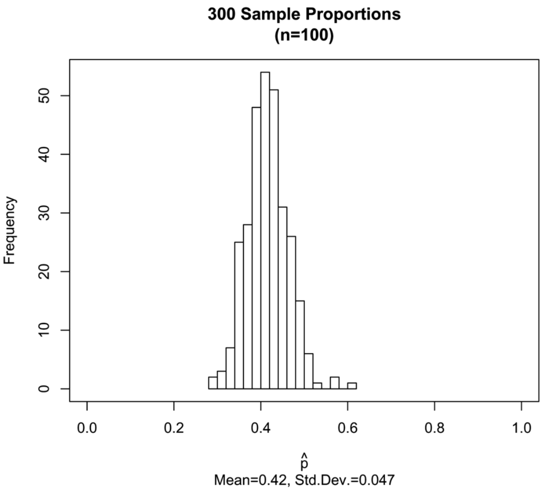
Example:

According to a *Time* poll conducted in June of 2008, 42% of registered voters believed that gay and lesbian couples should be allowed to marry.
Describe the sampling distribution of the sample proportion for samples of size n = 10, 50, 100.

Using simulations we get these:







Key Points:

- ***Shape:*** As the size of the sample, *n,* increases, the shape of the sampling distribution of the sample proportion becomes approximately normal.
- ***Center:*** The mean of the sampling distribution of the sample proportion equals the population proportion, *p*.
- ***Spread:*** The standard deviation of the sampling distribution of the sample proportion decreases as the sample size, *n*, increases.

Sampling Distribution of $\hat{p}$:

For a simple random sample of size *n* with population proportion *p*:

- The shape of the sampling distribution of $\hat{p}$ is approximately normal provided $np(1-p) \geq 10$.
- The mean of the sampling distribution (normal or not) of $\hat{p}$ is
$$\mu_{\hat{p}} = p$$
- The standard deviation of the sampling distribution (normal or not) of $\hat{p}$ is
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$
- The model on the previous slide requires that the sampled values are independent. When sampling from finite populations, this assumption is verified by checking that the sample size *n* is no more than 5% of the population size *N* ($n \leq 0.05N$).

Example: Describing the Sample Distribution of the Sample Proportion

According to a *Time* poll conducted in June of 2008, 42% of registered voters believed that gay and lesbian couples should be allowed to marry. Suppose that we obtain a simple random sample of 50 voters and determine which voters believe that gay and lesbian couples should be allowed to marry. Describe the sampling distribution of the sample proportion for registered voters who believe that gay and lesbian couples should be allowed to marry.

Is it Normal?

$\mu_{\hat{p}} =$

$\sigma_{\hat{p}} =$

Compute Probabilities of a Sample Proportion

<u>Example:</u> Compute Probabilities of a Sample Proportion
According to the Centers for Disease Control and Prevention, 18.8% of school-aged children, aged 6-11 years, were overweight in 2004.

(a) In a random sample of 90 school-aged children, aged 6-11 years, what is the probability that at least 19% are overweight?

(b) Suppose a random sample of 90 school-aged children, aged 6-11 years, results in 24 overweight children. What might you conclude?