9.2 Estimating a Population Mean

The **point estimate** for the population mean μ is the sample mean $\bar{x}$.

<u>Penny Example 1</u>: Computing a Point Estimate
Pennies minted after 1982 are made from 97.5% zinc and 2.5% copper. The following data represent the weights (in grams) of 17 randomly selected pennies minted after 1982.

2.46 2.47 2.49 2.48 2.50 2.44 2.46 2.45 2.49
2.47 2.45 2.46 2.45 2.46 2.47 2.44 2.45

Treat the data as a simple random sample.
Estimate the population mean weight of pennies minted after 1982.
Type these numbers into your TI-84 list (we'll use them later)

Solution:
The sample mean is:

The point estimate of μ is 2.464 grams.

t-Distribution

Now we find a confidence interval for the mean using a "t-score" instead of a "z-score". However, it's very similar in application.

<u>"Student's" t-Distribution</u>
Suppose that a simple random sample of size n is taken from a population. If the population from which the sample is drawn follows a ***normal distribution***, the distribution of

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

follows Student's t-distribution with $n - 1$ degrees of freedom where $\bar{x}$ is the sample mean and s is the sample standard deviation.

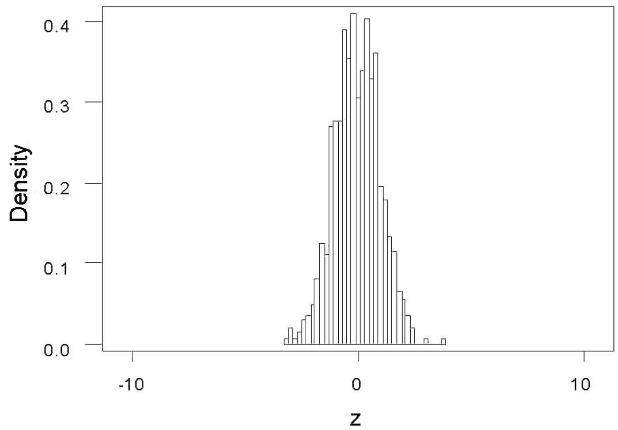
Example:

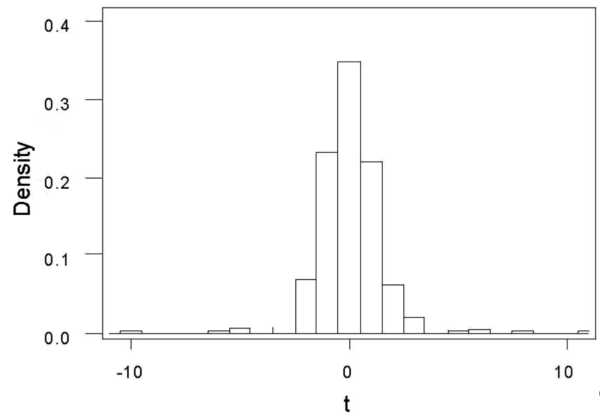
Suppose we obtain 1,000 simple random samples of size $n = 5$ from a normal population with $\mu = 50$ and $\sigma = 10$.

Determine the sample mean and sample standard deviation for each of the samples.

Compute $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ and $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ for each sample.

Draw a histogram for both z and t.





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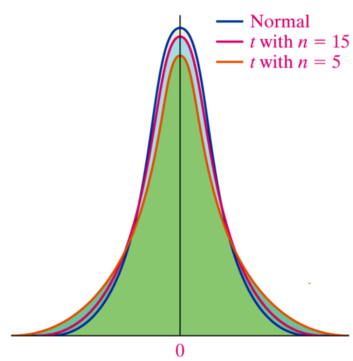
- Are both graphs symmetric and bell shaped?

- How is the spread of data different?

Conclusion: the t-statistic is closer to normal in this case.

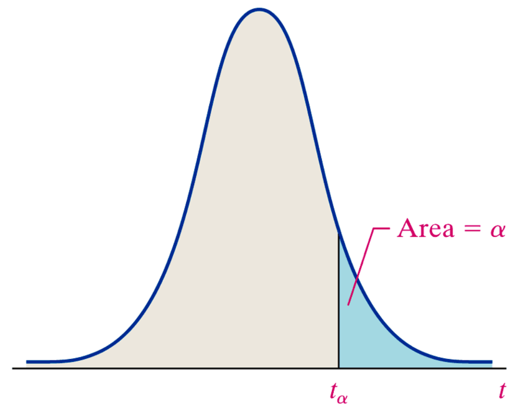
Properties of the t-Distribution

1. The t-distribution is different for different degrees of freedom.
2. The t-distribution is centered at 0 and is symmetric about 0.
3. The area under the curve is 1. The area under the curve to the right of 0 equals the area under the curve to the left of 0, which equals 1/2.
4. As t increases or decreases without bound, the graph approaches, but never equals, zero.
5. The area in the tails of the t-distribution is a little greater than the area in the tails of the standard normal distribution, because we are using s as an estimate of σ, thereby introducing further variability into the t- statistic.
6. As the sample size n increases, the density curve of t gets closer to the standard normal density curve. This result occurs because, as the sample size n increases, the values of s get closer to the values of σ, by the Law of Large Numbers.



Determining t-values

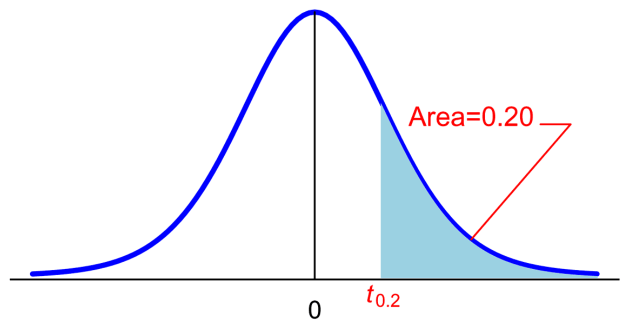
The notation t_α is the t-value such that the area under the standard normal curve to the right of α. This figure illustrates the notation.



<u>Example:</u> Finding *t*-values

Find the *t*-value such that the area under the *t*-distribution to the **right** of the *t*-value is 0.2 assuming 10 degrees of freedom.

That is, <u>find $t_{.80}$ with 10 degrees of freedom</u> because 80% of the area is to the left, and 20% is to the right (in the "tail").



t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$
one-tail	**0.50**	**0.25**	**0.20**
two-tails	**1.00**	**0.50**	**0.40**
df			
1	0.000	1.000	1.376
2	0.000	0.816	1.061
3	0.000	0.765	0.978
4	0.000	0.741	0.941
5	0.000	0.727	0.920
6	0.000	0.718	0.906
7	0.000	0.711	0.896
8	0.000	0.706	0.889
9	0.000	0.703	0.883
10	0.000	0.700	0.879
11	0.000	0.697	0.876

On TI-84: [2nd] – [DISTR] – [invT] – invT(.80,10)

Construct and Interpret Confidence Interval for the Population Mean

Constructing a Confidence Interval for the Population Mean

Provided (i.e. check these)

- Sample data come from a simple random sample or randomized experiment
- Sample size is small relative to the population size ($n \leq 0.05N$)
- The data come from a population that is normally distributed, or the sample size is large

A $(1-\alpha)\cdot 100\%$ confidence interval for μ is given by

Lower Bound: $\bar{x} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$

Upper Bound: $\bar{x} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$

Where $t_{\frac{\alpha}{2}}$ is the critical value with $n-1$ df (degrees of freedom).

Penny Example 2: Constructing a confidence interval

Construct a 99% confidence interval about the population mean weight (in grams) of pennies minted after 1982. Assume σ = 0.02 grams.

2.46	2.47	2.49	2.48	2.50	2.44	2.46	2.45	2.49
2.47	2.45	2.46	2.45	2.46	2.47	2.44	2.45	

- Step 1: Find the t -value $t_{\frac{\alpha}{2}}$ (on table or with invT on calculator)
- Step 2: Find the standard deviation s on your calculator
- Step 3: Find Lower bound:
  $\bar{x} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$
- Step 4: Find Upper Bound
  $\bar{x} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$
- State Solution:

  We are 99% confident that the mean weight of pennies minted after 1982 is between

  ____________________ and ____________________grams.

  On TI-84, use: **[STAT] – [Tests] – [Tinterval]**

Find the Sample Size Needed to Estimate the Population Mean within Margin of Error

The sample size required to estimate the population mean, μ, with a level of confidence $(1-\alpha)\cdot 100\%$ with a specified margin of error, E, is given by

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot s}{E}\right)^2$$

where n is rounded up to the nearest whole number.

Penny Example 3: Determining Sample Size

Back to the pennies. How large a sample would be required to estimate the mean weight of a penny manufactured after 1982 within 0.005 grams with 99% confidence? Assume $\sigma = 0.02$.

- Step 1: Calculate $z_{\frac{\alpha}{2}}$
- Step 2: Find sample standard deviation s (using a known standard deviation.)
- Step 3: Locate Margin of Error
- Step 4: Calculate n

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot s}{E}\right)^2$$