

# Chapter 1:

# Tools of Algebra

Using algebraic formulas requires skill with all types of numbers (negatives, fractions and decimals), an understanding of measurements for lengths and angles, familiarity with units (such as feet,  $\text{mm}^2$ ,  $\text{in}^3$ ), and a firm grasp of the order of operations. We are at the trail head and this is an equipment check. You should notice that you already have much of what you will need. This chapter will refresh your memory, deepen your grasp of the basics, and introduce a few new concepts that you will find useful on the trail.



## 1.1: Operations with Real Numbers

The study of Algebra must begin with a review of the basic operations (+, -, x, ÷) on the numbers that show up in the real world (negatives, fractions, and decimals). Stirred together, they are appropriately called real numbers (such as 5, -3.62,  $-\frac{5}{8}$ ). Recall the facts for negative numbers and fractions:

### Negative Number Facts for Multiplication:

1.  $(+)(+) = +$
2.  $(+)(-) = -$
3.  $(-)(+) = -$
4.  $(-)(-) = +$

### Fraction Facts:

1.  $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
2.  $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$
3.  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
4.  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

**Side note:**  $-(-7)$  is quite logically  $+7$  since  $-(-7)$  could represent taking away a debt of \$7. Taking away debt is always positive.

Let's review multiplication with decimals first:

### Example 1.1.1: Multiplying decimals with negatives

Simplify:  $-4.2 \cdot (-6.3)$

#### Solution:

Stop and think: The answer will be positive and it is always a good idea to estimate first:  $-4 \cdot (-6) = 24$ , so we can expect the exact answer to be larger than 24.

**Final Answer:** 26.46

Apply the negative number facts to an example with fractions:

### Example 1.1.2: Dividing fractions with negatives

Simplify:  $6\frac{2}{3} \div -5\frac{1}{4}$

#### Solution:

Stop and think: The answer will be negative and it is always a good idea to estimate first:  $6 \div 5$  will be a little larger than 1. We can ignore the sign during the work and just include it in the answer:

$$6\frac{2}{3} \div 5\frac{1}{4}$$

$$\frac{20}{3} \div \frac{21}{4}$$

$$\frac{20}{3} \times \frac{4}{21}$$

$$\frac{80}{63}$$

$$1\frac{17}{63}$$

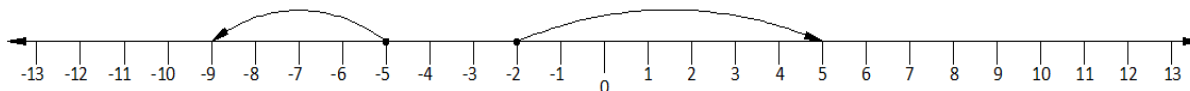
$6 = \frac{18}{3}$  and  $\frac{18}{3} + \frac{2}{3} = \frac{20}{3}$ ;  $5 = \frac{20}{4}$  and  $\frac{20}{4} + \frac{1}{4} = \frac{21}{4}$   
 invert (flip) the second fraction then multiply  
 fractions are multiplied straight across  
 $\frac{80}{63} = \frac{63}{63} + \frac{17}{63} = 1 + \frac{17}{63}$

**Final Answer:**  $-1\frac{17}{63}$

Addition and subtraction with negative numbers is simple if you think of the sign as a symbol for direction. A football team could gain 12 yards (+12) or lose 12 yards (-12). The 12 tells us the distance traveled, while the sign tells us the direction. It is an unfortunate but unavoidable fact that all of life's experiences are not positive. A carpenter can cut a rafter that is off by  $\frac{1}{8}$  inch.  $+\frac{1}{8}$  represents  $\frac{1}{8}$ " too long and  $-\frac{1}{8}$  represents  $\frac{1}{8}$ " too short. Operations with negative numbers are easy to picture if you use a number line to represent direction. Moving right is positive, left is negative.

Examples:

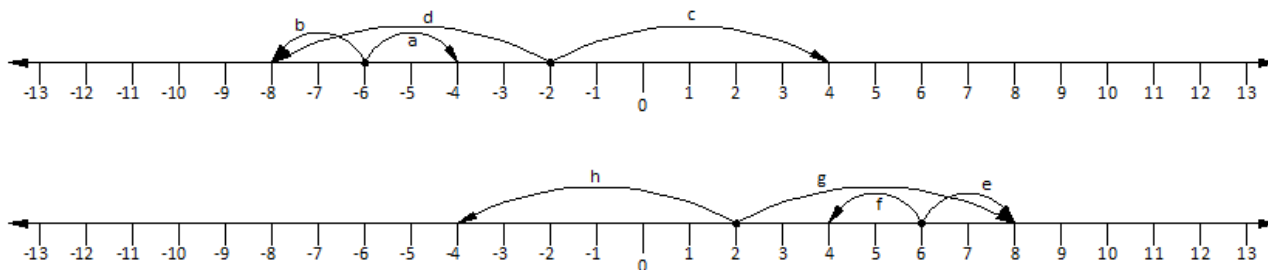
1.  $-5 - 4 = -9$ : begin at -5 then move left 4 to -9
2.  $-2 - (-7) = 5$ : begin at -2 and move right 7 to 5 (right because a  $(-)(-)$  is a positive)



It is interesting to note that, despite the variety signs add to a problem involving a 6 and a 2,

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| a) $-6 + 2$ | b) $-6 - 2$ | c) $-2 + 6$ | d) $-2 - 6$ |
| e) $6 + 2$  | f) $6 - 2$  | g) $2 + 6$  | h) $2 - 6$  |

the absolute value of the answer (the value of the number ignoring the sign) must either be 8 or 4 in every case. Study the illustration below:



Subtracting and adding with negative numbers is as simple as beginning at the first number then moving the distance and direction specified by the second number. The process can be understood so that you do not need a list of rules and steps to follow for each different case.

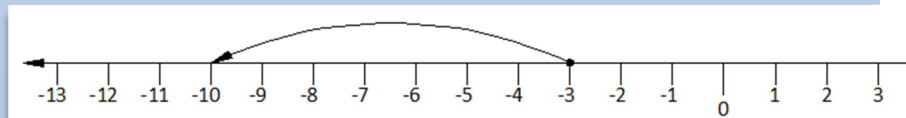
Let's work an addition example:

### Example 1.1.3: Adding negative numbers

Simplify:  $-3 + (-7)$

#### Solution:

Start at -3, and move left 7.



**Final Answer:** -10

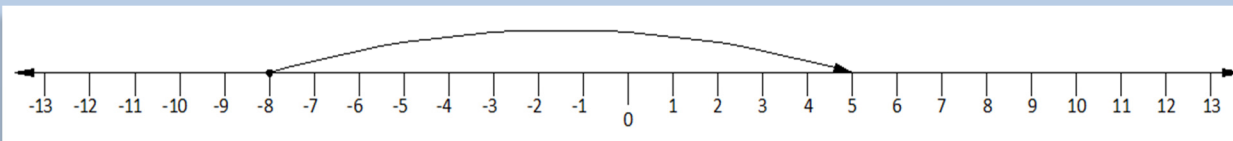
Let's work a subtraction example:

### **Example 1.1.4: Subtracting negative numbers**

Simplify:  $-8 - (-13)$

#### **Solution:**

Start at -8, and move right 13.



**Final Answer:** 5

Notice that in example 1.1.3 we ended up adding the 3 and 7 since we started by moving left to -3 then left again 7 more. In example 1.1.4 we subtracted the 13 and 8 since we started by moving left to -8 then 13 back to the right. We could proceed to a rule here but you won't need it if you have a picture of what is happening on the number line.

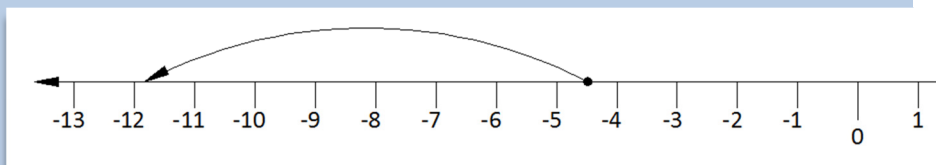
Let's extend our understanding to include decimals:

### **Example 1.1.5: Subtracting decimals with negatives**

Simplify:  $-4.548 - 7.289$

#### **Solution:**

Start at -4.548 and move left 7.289. You will add the numbers and be on the negative side of 0.



**Final Answer:** -11.837

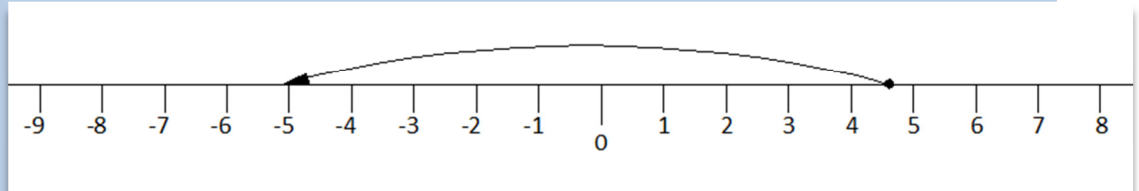
Let's extend our understanding to include fractions:

### Example 1.1.6: Adding fractions with negatives

Simplify:  $4\frac{5}{8} + (-9\frac{3}{4})$

**Solution:**

Start at  $4\frac{5}{8}$  and move left  $9\frac{3}{4}$ . You will subtract the numbers and be on the negative side of 0.



$$9\frac{3}{4} - 4\frac{5}{8}$$

set up the subtraction problem

$$9\frac{6}{8} - 4\frac{5}{8}$$

find a common denominator for 4 and 8

$$5\frac{1}{8}$$

simplify

$$-5\frac{1}{8}$$

include the negative sign since the number is on the negative side of zero

**Final Answer:**  $-5\frac{1}{8}$

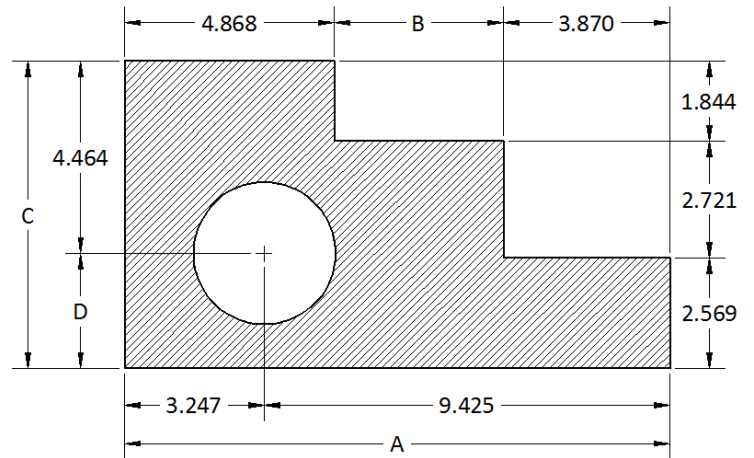
## Section 1.1: Operations with Real Numbers



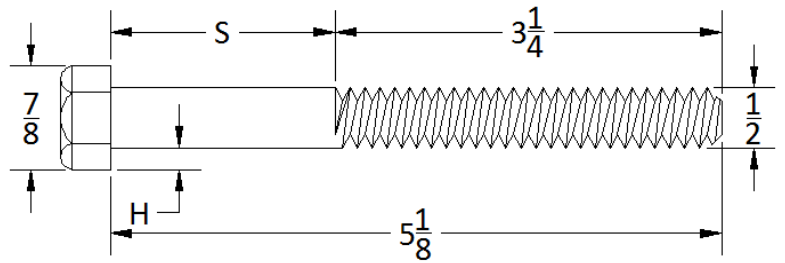
1. A carpenter needs a board that is  $79\frac{5}{8}$ " long that she will cut from a board is  $96\frac{1}{4}$ " long. If the saw blade has a  $\frac{3}{16}$ " kerf blade (kerf is the thickness of wood that the blade removes with a cut), calculate the length of the left over piece.



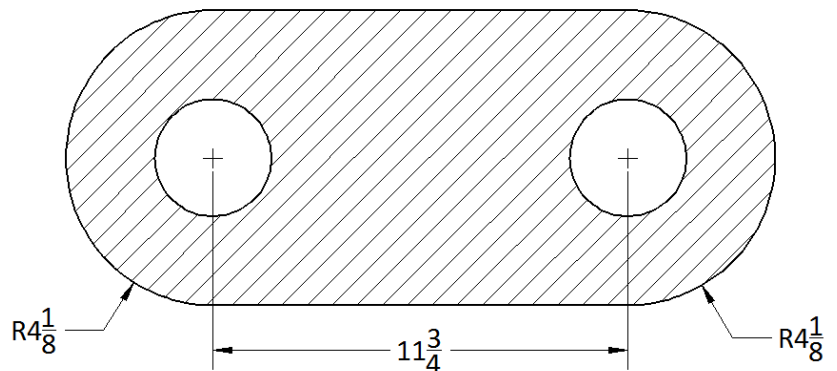
2. Calculate the missing dimensions A, B, C, and D. Measurements are in inches.



3. Calculate the length of the non-threaded portion of the bolt (S). Calculate the size of the overhang (H). Measurements are in inches.

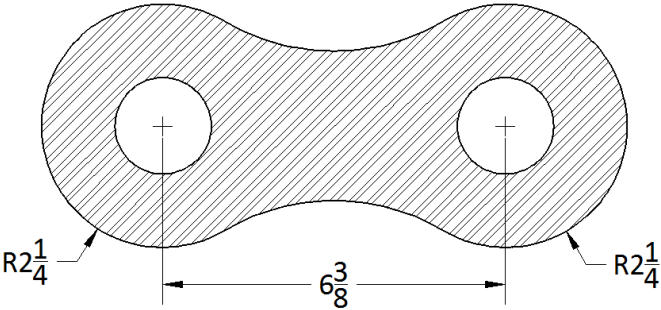


4. If the drawing of the part below is scaled up by a multiple of five, find the new dimensions. Measurements are in inches. Note: An R in drafting denotes the radius of a circle.

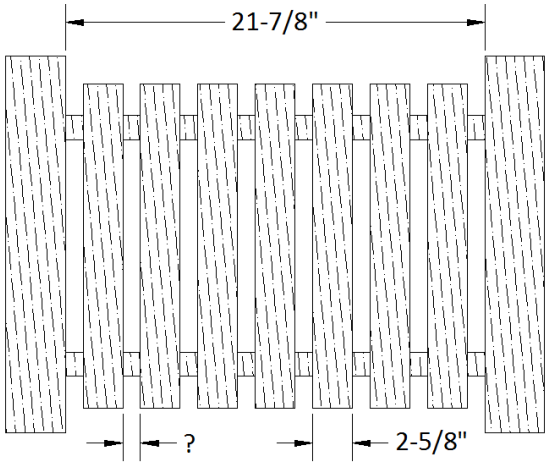




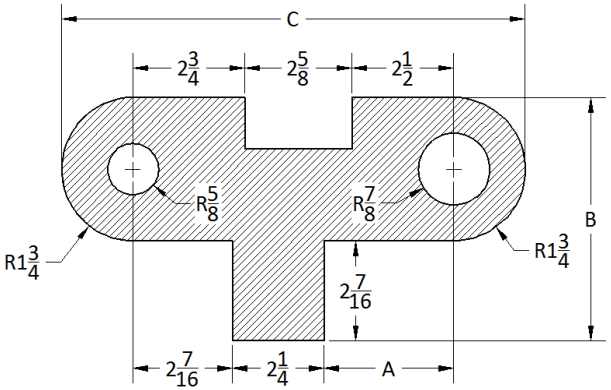
5. If the drawing of the link below is scaled up by a factor of seven, find the new dimensions for the overall width and height. Measurements are in inches. Note: An R in drafting denotes the radius of a circle.



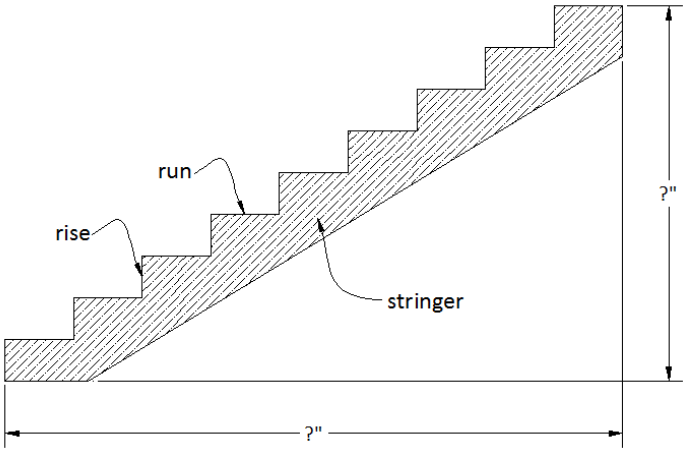
6. a) A fence is to have seven boards between two posts so that the space between each board is the same. Calculate the distance between each board. Answer as a fraction.
- b) If the carpenter decided the distance between boards was too large, would it be possible to add an extra board by decreasing the space between each board? Explain. Note: dashes are commonly placed between whole numbers and fractions so  $21 \frac{7}{8}$  is not mistaken for  $\frac{217}{8}$ .



7. Calculate the missing dimensions A, B, and C. Measurements are in inches. Note: An R in drafting denotes the radius of a circle.

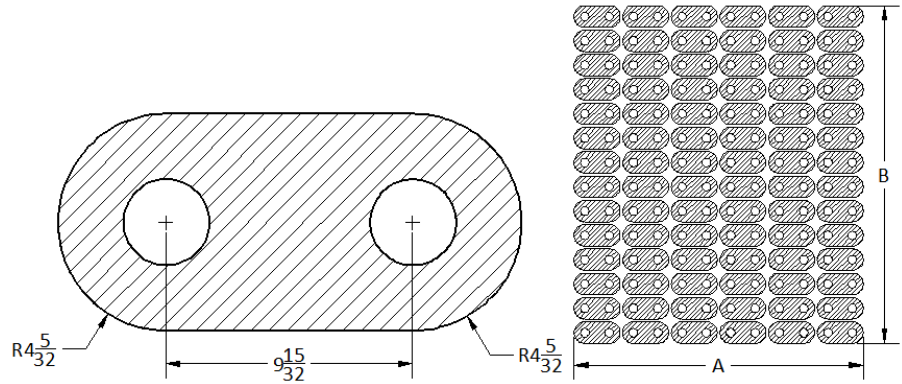


8. A stair stringer is cut with a  $7 \frac{9}{16}$  rise and a  $11 \frac{5}{8}$  run. Code stipulates that all rises must be the same height and all runs must be the same length to avoid being a trip hazard. Calculate the total rise and total run for the stringer with nine steps.

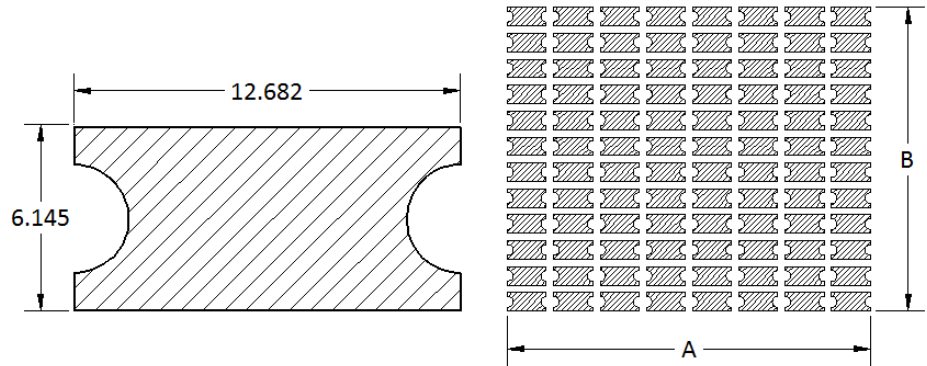


Chapter 1

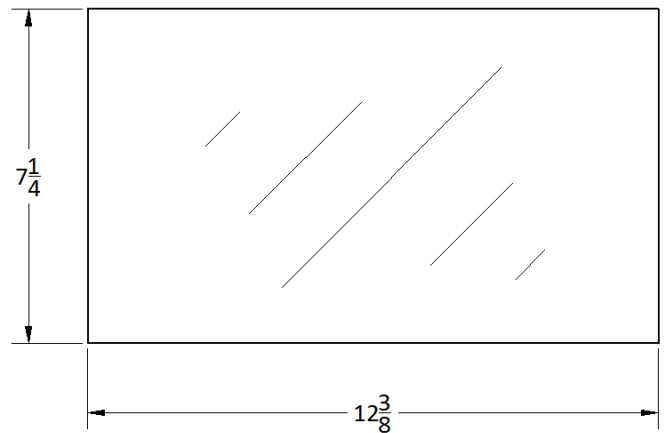
9. A manufacturer wants to layout the parts on a sheet of steel in 14 rows and 6 columns with a  $\frac{1}{2}$ " space between each row and column. Calculate the width (A) and length (B) of the sheet that will be needed to produce the parts. How many parts will this design produce? Measurements are in inches.



10. A manufacturer wants to lay out the parts on a sheet of plastic in 12 rows and 8 columns with a .15-inch space between each row and column. Calculate the width (A) and length (B) of the sheet of that will be needed to produce the parts. How many parts will this design produce? Measurements are inches.

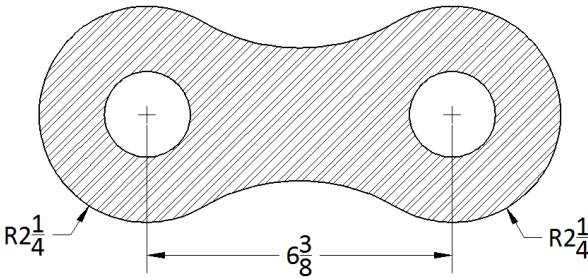


11. Calculate the area and perimeter of a rectangular sheet of glass that is  $12\frac{3}{8}$  x  $7\frac{1}{4}$ , answer as a fraction.

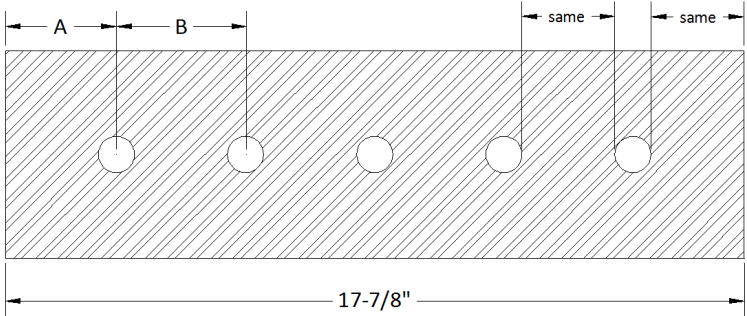




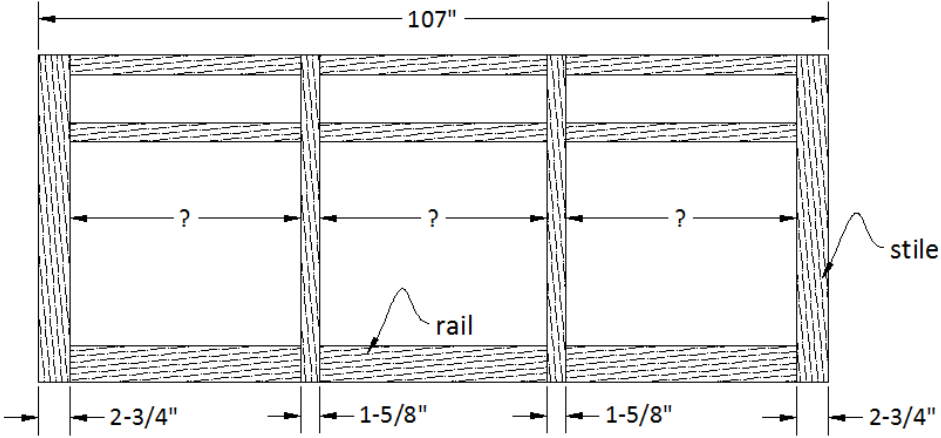
12. If the drawing of the link below is scaled down by a factor of four (divide by four), find the new dimensions for the overall width and height. Measurements are in inches.



13. A steel plate is to have five  $\frac{7}{8}$ " diameter holes drilled so that the distance between the edges of each hole and the distance between the edge of each hole and the edge of the plate is the same. Calculate the distance from the edge of the plate to the center of the first hole (A), then the distance between the center of each hole (B). Answer as a fraction.



14. The face frame of a cabinet is made of vertical stiles and horizontal rails. Calculate the width of the rails in the design below so that all nine rails are the same length.





15. In electronics large or small quantities are written using metric prefixes. For example:

$$3.2 \mu\text{A} = 3.2 \times 10^{-6} \text{ A} = 3.2 \times .000001 \text{ A} = .0000032 \text{ A}$$

$$8.3 \text{ k}\Omega = 8.3 \times 10^3 \Omega = 8.3 \times 1000 \Omega = 8300 \Omega$$

Notice that the power of 10 moves the decimal left or right according to the sign of the number.

Expand the following without prefixes:

a) 6.32 GV

b) 9.8 nA

c) .47  $\mu\text{F}$

d) 14.2 MW

Common Electronics Units	
Unit	Symbol
W	Watt
$\Omega$	Ohm
V	Volt
A	Amp
F	Farad
H	Henry
Hz	Hertz

Table of Common Metric Prefixes			
Metric Prefix	Symbol	Power of 10	Factor
Tera	T	$10^{12}$	1,000,000,000,000
Giga	G	$10^9$	1,000,000,000
Mega	M	$10^6$	1,000,000
kilo	k	$10^3$	1,000
milli	m	$10^{-3}$	.001
micro	$\mu$	$10^{-6}$	.000001
nano	n	$10^{-9}$	.000000001
pico	p	$10^{-12}$	.000000000001

16. Scientific Notation is a number between 1 and 10 times a power of ten.



Engineering Notation is a modification of Scientific Notation where the number is between 1 and 1000 and the power of 10 is always a multiple of three, making it easy to write the number using a metric prefix.

Consider the two examples in the table and fill in the rest using the table of common metric prefixes.

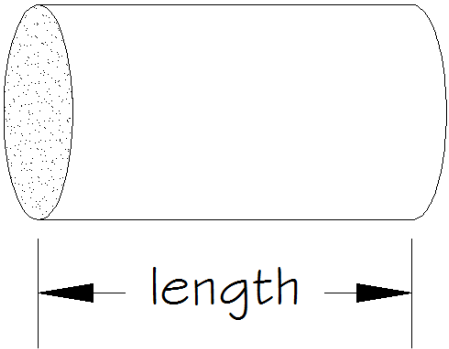
Standard Form	Scientific Notation	Engineering Notation	Metric Prefix
12,300 $\Omega$	$1.23 \times 10^4$	$12.3 \times 10^3$	12.3 k $\Omega$
.000072 A	$7.2 \times 10^{-5}$	$72 \times 10^{-6}$	72 $\mu\text{A}$
480,000 Hz			
.00043 V			
7,500 W			
.000000042 F			
62,000,000 H			

Table of Common Metric Prefixes			
Metric Prefix	Symbol	Power of 10	Factor
Tera	T	$10^{12}$	1,000,000,000,000
Giga	G	$10^9$	1,000,000,000
Mega	M	$10^6$	1,000,000
kilo	k	$10^3$	1,000
milli	m	$10^{-3}$	.001
micro	$\mu$	$10^{-6}$	.000001
nano	n	$10^{-9}$	.000000001
pico	p	$10^{-12}$	.000000000001

17. Consider a rod being cut to a length. The length can never be perfect and so must be cut for accuracy based on a desired tolerance. Tolerance is the amount that a measurement can vary above or below a design size.



Design Length	Tolerance	Maximum Length	Minimum Length
17.317 in	$\pm .034$ in		
18.12 cm	$\pm .15$ cm		
$26 \frac{5}{8}$ in	$\pm \frac{1}{32}$ in		
3.2 meters	$\pm .06$ cm	cm	cm

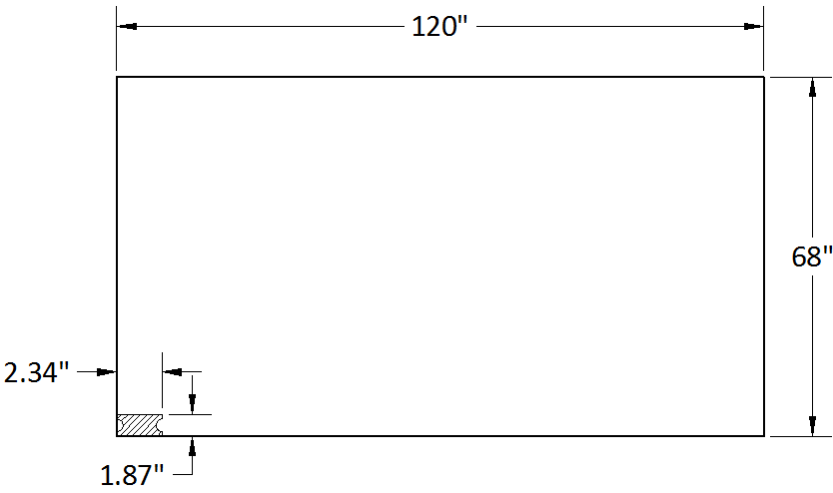


18. Calculate the difference between the design size and the actual size in the chart. Actual sizes that are less than the design size should be denoted with negative numbers.



Design Size	Actual Size	Difference
$14 \frac{3}{16}$ in	$13 \frac{29}{32}$ in	
18.43 mm	18.38 mm	
17.972 in	18.013 in	

19. **Challenge Problem:** A manufacturer wants to lay out parts on a sheet of steel that is 120" x 68". If the part is 2.34" wide and 1.87" tall, calculate the number of rows and columns that will fit if there is a .6" space between each part. How many parts will this design produce? Note: the drawing is not to scale.





20. **Challenge Problem:** A manufacturer wants to lay out parts on a sheet of steel that is 108" x 60". If the part is  $2\frac{5}{16}$ " wide and  $1\frac{1}{8}$ " tall, calculate the number of rows and columns that will fit if there is a  $\frac{3}{4}$ " space between each part. How many parts will this design produce? Note: the drawing is not to scale.

