# 1.3: Ratio, Proportion and Percent

Proportions are one of the simplest and most powerful tools that math has to offer. I recently replaced the ridge cap on a colleague's roof. The lineal footage to be replaced measured 256 feet. How many boxes to order? My problem was easily solved making a proportion by setting 2 ratios equal to each other. Each box of ridge cap contained 20 lineal feet so: $\frac{1 \text{ box}}{20 \text{ ft}} = \frac{? \text{ boxes}}{256 \text{ feet}}$. The answer was 12.8 boxes so I ordered 13 boxes. Each box of ridge cap contained 30 individual ridge caps which are attached with 2 nails each. I would need 60 nails per box times 13 boxes, so 390 nails. Counting this many nails would take an unreasonable amount of time so the salesman used a proportion based on the table to sell me the nails by weight. I used $1\frac{1"}{2}$ nails so:

$\frac{1 \text{ lb}}{180 \text{ nails}} = \frac{? \text{ lbs}}{390 \text{ nails}}$. He sold me 2.2 lbs.

| Length | Gauge | Head | Nails/Lb. |
|---|---|---|---|
| 7/8" | 11 | 3/8" | 272 |
| 1" | 11 | 3/8" | 250 |
| 1-1/4" | 11 | 3/8" | 202 |
| 1-1/2" | 11 | 3/8" | 180 |
| 1-3/4" | 11 | 3/8" | 156 |
| 2" | 11 | 3/8" | 136 |

The power of the proportion lies in establishing one ratio relating two quantities you are interested in, then equating that to another. I found that I was able to install 24 feet of ridge cap in 32 minutes. I wanted to know if I would be done in time to pick up my daughter from school. A proportion allowed me to predict how long it would take to finish the job.

## Example 1.3.1: Roofing application

How much time will it take to install 256 feet of ridge cap if the first 24 feet are installed in 32 minutes?

**Solution:**

Set up a proportion: $\frac{32 \text{ min}}{24 \text{ ft}} = \frac{? \text{ min}}{256 \text{ ft}}$.

$\frac{32}{24} = \frac{t}{256}$ rewrite without the units

$32 \bullet 256 = 24 \bullet t$ We will discuss solving equations more carefully in chapter 2, for now, cross multiplying eliminates the fractions. The dot (•) is a common symbol for multiplication in algebra since the usual symbol (x) is also used as a letter.

$8192 = 24t$ simplify

$t \approx 341$ divide both sides by 24

**Final Answer:** total time for the job 341 minutes. I had already worked for 18 minutes so that left about 323 minutes.

**Side note 1:** I didn't factor in that I am 44 years old I cannot bend over for $5\frac{1}{2}$ hours straight. I was late.

**Side note 2:** I only estimated this since the situation did not call for a great deal of accuracy, and more importantly, carrying your calculator with you in life is universally regarded as nerdy.

Consider another example:

## Example 1.3.2: Sloping concrete for drainage

Concrete contractors typically slope garage floors at ¼" per foot so that water will drain off. If the floor is 26 feet long, find the total amount of fall or drop in the floor.

**Solution:**

Set up a proportion: $\frac{\frac{1}{4}\text{ in}}{1\text{ ft}} = \frac{?\text{ in}}{26\text{ ft}}$.

$\frac{\frac{1}{4}}{1} = \frac{x}{26}$ without the units

$26 \bullet \frac{1}{4} = 1 \bullet x$ cross multiply

$6\frac{1}{2} = x$ simplify

**Final Answer:** The total fall in the floor will be $6\frac{1}{2}"$.

Another excellent use for proportions arises when converting decimal calculations to fractions in the standard system of measurement. This was briefly considered in section 1.2.

## Example 1.3.3: Roof slope application

A roof slopes with a rise of 7 and run of 12. Find the rise accurate to the nearest 16th of an inch for a run of $118\frac{5}{8}"$.

**Solution:**

$\frac{7}{12} = \frac{x}{118.625}$ both rises on the top of the fraction, both runs on the bottom

$7 \bullet 118.625 = 12 \bullet x$ cross multiply

$830.375 = 12x$ simplify

$x \approx 69.198$ divide both sides by 12

Although this answer is correct, it is not measurable with a typical ruler.
We need to figure out how many 16ths are in .198, which can also be solved with a proportion.

$\frac{.198}{1} = \frac{?}{16}$ how many 16ths equal .198?

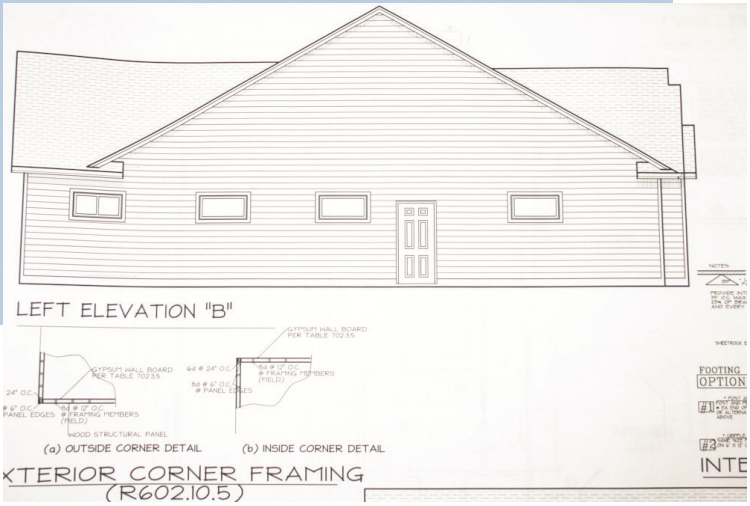
$? \approx 3.2$ cross multiply (.198" is approximately $\frac{3}{16}"$)

**Final Answer:** The roof will rise approximately $69\frac{3}{16}"$.

A final application is found in reading plans:

## Example 1.3.4: Scale drawing application

A manufacturer is drawing a plan scaled at $\frac{1"}{8} = 1"$ (meaning the plan is drawn 1/8 the size of the real part). If the part measures 35.348 inches, find the length of the measure on the plan to the nearest 10th of an inch.




**Solution:**

$\frac{\frac{1}{8}}{1} = \frac{x}{35.348}$ set up a proportion

$x \approx 4.4"$ cross multiply

**Final Answer:** Draw the part approximately 4.4".

A fraction can have any number in the denominator. A percent is simply a fraction with a denominator of 100. Percents are convenient for comparison because as the name (per-cent) implies, they are always per 100 or $\frac{?}{100}$. The symbol "%" is used in place of the fraction for convenience. If you think about the word percent it should be evident that 32% = $\frac{32}{100}$ = .32. When you see "37%", think "37 hundredths". When you encounter a fraction like $\frac{2}{5}$, keep in mind that changing it to percent means the same thing as changing it to hundredths. Thus, $\frac{2}{5} = \frac{40}{100}$ = 40%.

In example 1.3.1, I noted that I had completed $\frac{24 \text{ ft}}{256 \text{ ft}}$ of my roofing job. If this fraction is changed to a percent, it is easier to have a feel for how much of the job I have completed. One method for changing to a percent is to use a proportion. We must change $\frac{24}{256}$ to hundredths, so we write $\frac{24}{256} = \frac{x}{100}$. Cross-multiplying, we see that 2400 = 256x, so x ≈ 9. From this, we can conclude that $\frac{24}{256}$ is about 9 hundredths, so I had completed about 9% of the job.

A second method for changing a fraction to a percent involves the fraction bar in $\frac{24}{256}$, indicating that 24 is divided by 256. The fraction $\frac{24}{256}$ = .09375 which rounds to .09 or 9%.

Percents are regularly used in the financial aspects of trade.

## Example 1.3.5: Costing out a job

A welder agrees to build a trailer for cost plus 18%. Calculate the total bill for the job if the costs totaled $1,460.

**Solution:**

| | |
|---|---|
| 18% of 1,460 | find the amount to add to the cost |
| $\frac{18}{100}$ • 1,460 | 18 percent = 18 hundredths and "of" is a word implying multiplication |
| .18 • 1,460 | 18 hundredths in decimal form |
| 262.8 | simplify |
| 1,460 + 262.8 | costs plus the added 18% |

**Final Answer:** He will expect to be paid $1,460 + $262.80 = $1,722.80.

## Example 1.3.6: Applying a discount

A lumber bill arrives with a note that you can subtract 7% if you pay on time. Calculate the amount you should pay if the bill is for $17,654.

**Solution:**

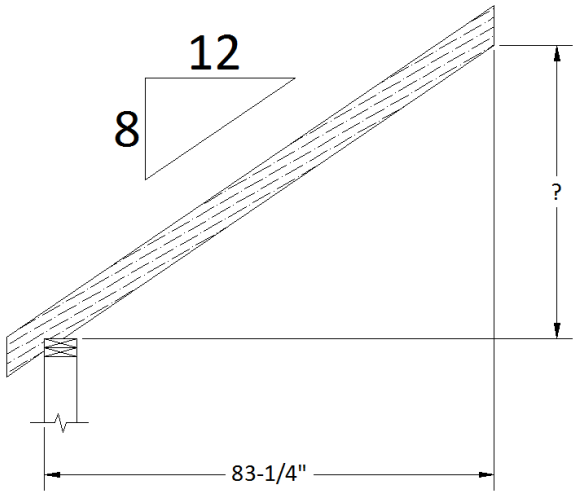
| | |
|---|---|
| 7% of 17,654 | find the amount to subtract from the bill |
| .07 • 17,654 | "of" means multiply and 7% as a decimal |
| 1,235.78 | simplify |
| 17,654 – 1,235.78 | bill amount minus the 7% discount |

**Final Answer:** You would pay $17,654 – $1,235.78 = $16,418.22.

Note: A wonderful shortcut: Saving 7% would mean that you are paying 93%. And .93 x 17,654 = 16,418.22, which is the same result.

## Section 1.3: Ratio, Proportion and Percent

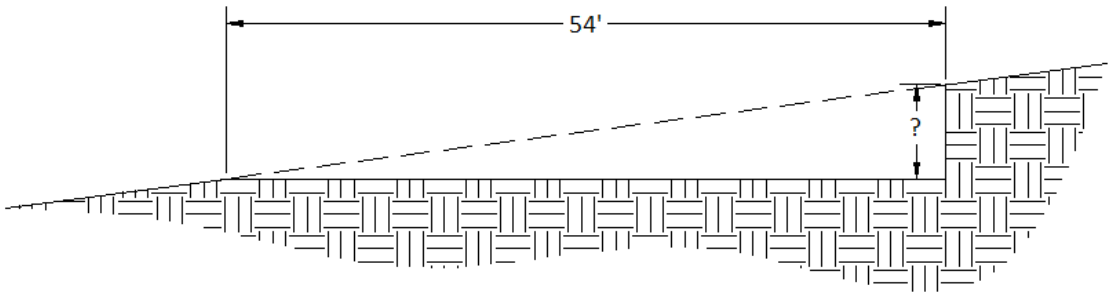
1. Use a proportion to calculate the height that a rafter will reach above the wall that it rests on if it is sloped at 8/12.



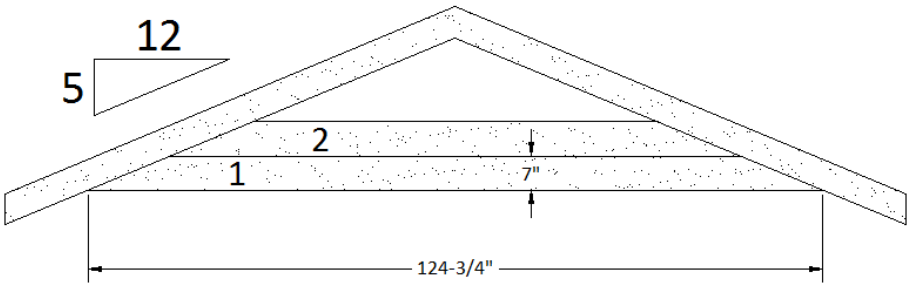

2. A slope of ¼" per foot is commonly used by concrete companies to ensure the water doesn't pool. Use a proportion to calculate the amount of fall that a garage floor should have if it is 18'-6" long. Note: 18'-6" means 18 feet plus 6 inches.

3. Plumbers use a slope of ¼" per foot to ensure the proper flow in waste pipes. Use a proportion to calculate the amount of fall that a pipe should have if it is 31'-3" long.

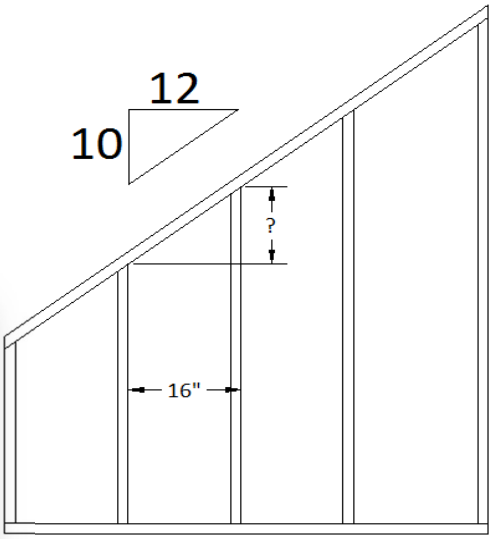
4. The slope of a hill is 1/8. If an excavator is making a level cut for a house pad that is 54', calculate the height of the bank that will result at the uphill side of the cut. Answer in inches.



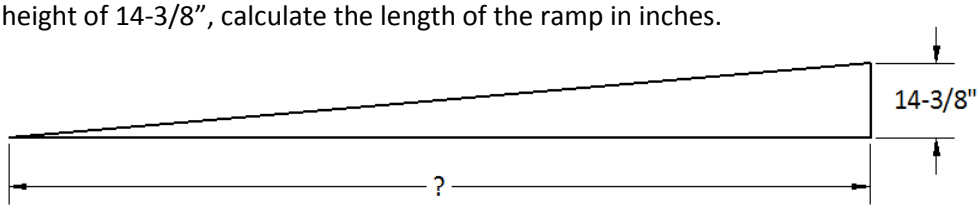
5. In a gable, each trapezoidal piece of lap siding is shorter than the one below by the same amount. A proportion can be used to calculate that amount, allowing a carpenter to cut the pieces without taking measurements. The bottom of each piece of siding is placed 7 inches above the bottom of the piece below. Use the roof's slope of 5/12 and the long point to long point measurement of the first piece, to set up a proportion and calculate the long point to long point measurement of the 2nd piece. Round your answer to the nearest 16th of an inch.




6. Studs in a framed wall are placed 16 inches apart. A sloped wall presents a challenge in that each stud must be cut to a different length. If the top plate has a slope of 10/12, set up a proportion to calculate the difference in length rounded to the nearest 16th of an inch.




7. The ADA (American Disabilities Act) specifies that a ramp must have a slope of 1/12. If a ramp must attain a height of 14-3/8", calculate the length of the ramp in inches.



8. A well produces 18 gallons per minute (GPM). Find the time it will take to fill a 1320 gallon pool rounded to one decimal place.

9. A car gets 32 miles per gallon (MPG). Find the number of gallons required to travel 857 miles rounded to one decimal place.

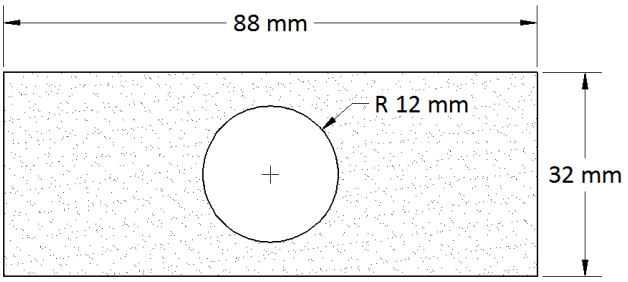
10. A car travels at 68 miles per hour (MPH). Find the time necessary to travel 487 miles rounded to one decimal place.

11. A shaper has a feed rate of 14 feet per minute. Find the time necessary to mill 1700 feet of trim rounded to one decimal place.

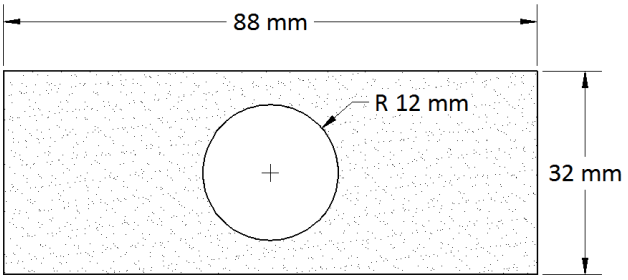
12. The lumber bill for a job totaled $18,400. If a 7% discount was then applied, find the discounted cost for the lumber.

13. If a contractor bills out his work at cost plus 15% (cost + 15% of the cost = total amount), figure the total bill if his costs were $1,340.

14. If the part below is scaled up 18%, find all three new dimensions.

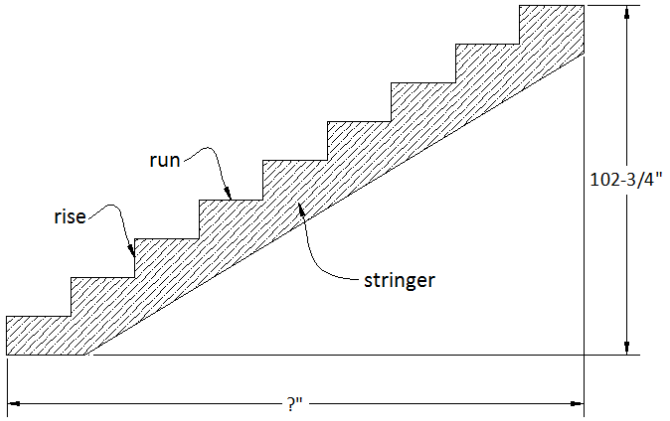



15. If the part below is scaled down 24%, find all three new dimensions.

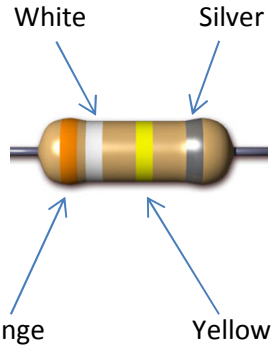



16. The Manual of Steel Construction states the tolerance for weight variation is plus or minus 2.5%. A 28 foot length of structural tubing is designed to weigh 60.75 pounds per foot. Calculate its weight and the heaviest and lightest it can be inside of the 2.5% tolerance. Round your answer to the nearest pound.

17. A stair stringer is to be cut with a ratio of rise to run of $7\frac{1"}{8}$ to $11\frac{1"}{2}$. If the total rise is $102\frac{3"}{4}$, calculate the total run rounded to the nearest 16th of an inch. Note: The picture is only to help understand the problem; the actual stringer will have more than nine steps.

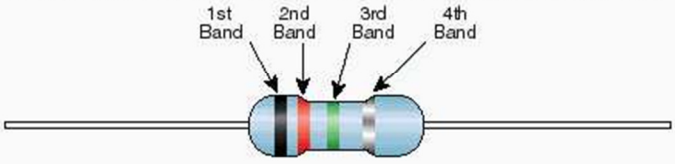



18. Resistors are common in electrical circuits and used to resist the flow of electricity. The colors identify the value of the resistor. Example:




Orange = 3, White = 9 and Yellow = $10^4$.

**Standard EIA Color Code Table** 4 Band: ±2%, ±5%, and ±10%




| Color | 1st Band (1st figure) | 2nd Band (2nd figure) | 3rd Band (multiplier) | 4th Band (tolerance) |
|---|---|---|---|---|
| Black | 0 | 0 | $10^0$ | |
| Brown | 1 | 1 | $10^1$ | |
| Red | 2 | 2 | $10^2$ | ±2% |
| Orange | 3 | 3 | $10^3$ | |
| Yellow | 4 | 4 | $10^4$ | |
| Green | 5 | 5 | $10^5$ | |
| Blue | 6 | 6 | $10^6$ | |
| Violet | 7 | 7 | $10^7$ | |
| Gray | 8 | 8 | $10^8$ | |
| White | 9 | 9 | $10^9$ | |
| Gold | | | $10^{-1}$ | ±5% |
| Silver | | | $10^{-2}$ | ±10% |

The design value of the resistor is 39 x $10^4$ or 390000 Ω or 390 kΩ. The silver band represents a tolerance of ± 10%. 10% of 390 = 39, so the minimum value is 390 – 39 or 351 kΩ and the maximum value is 390 + 39 or 429 kΩ.

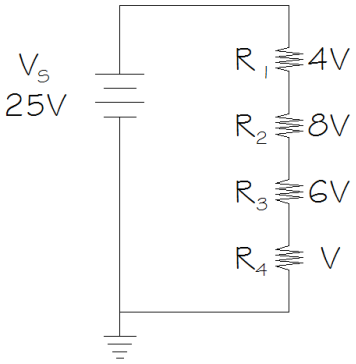
Find the design value, minimum value and maximum value for each of the resistors. Answer in kΩ.

a) 1st band = red, 2nd band = green, 3rd band = orange, 4th band = gold

b) 1st band = gray, 2nd band = blue, 3rd band = yellow, 4th band = red

c) 1st band = violet, 2nd band = white, 3rd band = red, 4th band = silver

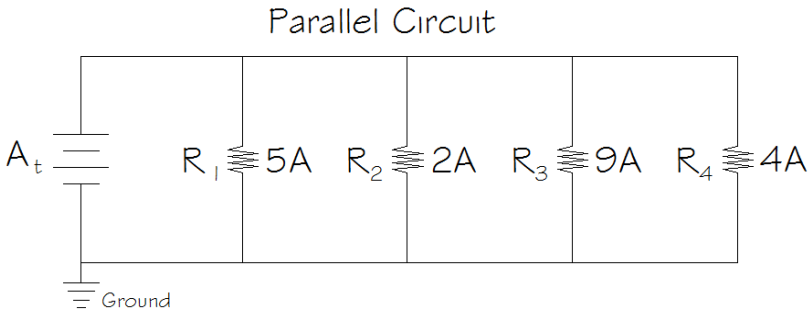
19. Kirchhoff's Voltage Law (KVL) states that $V_t = V_1 + V_2 + V_3 + V_4 \ldots$ for a series circuit.




a) Calculate the voltage at $R_4$ in the series circuit using KVL.
b) Calculate the percentage of the total voltage at $R_2$.
c) Calculate the percentage of the total voltage across $R_1$ and $R_2$ combined.

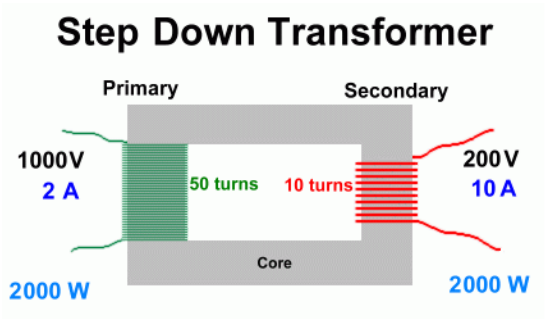
20. Kirchhoff's Current Law (KCL) states that $I_t = I_1 + I_2 + I_3 + I_4 \ldots$ for a parallel circuit. Current is measured in amps but abbreviated with an I.

a) Calculate the total current $I_t$ in the parallel circuit using KCL.
b) Calculate the percentage of the total current at $R_3$.
c) Calculate the percentage of the total current across $R_1$ and $R_2$ combined.




21. Transformers are used in electronics to step up to a higher voltage or step down to a lower voltage. Step down transformers are often located at the top of telephone poles to reduce the overhead voltage to a level suitable for a home.

The Step Down Transformer diagram shows 1000 V on the primary side stepping down to 200 V on the secondary side. As the diagram illustrates, it is done with the number of turns or windings. The following ratio describes the relationship:



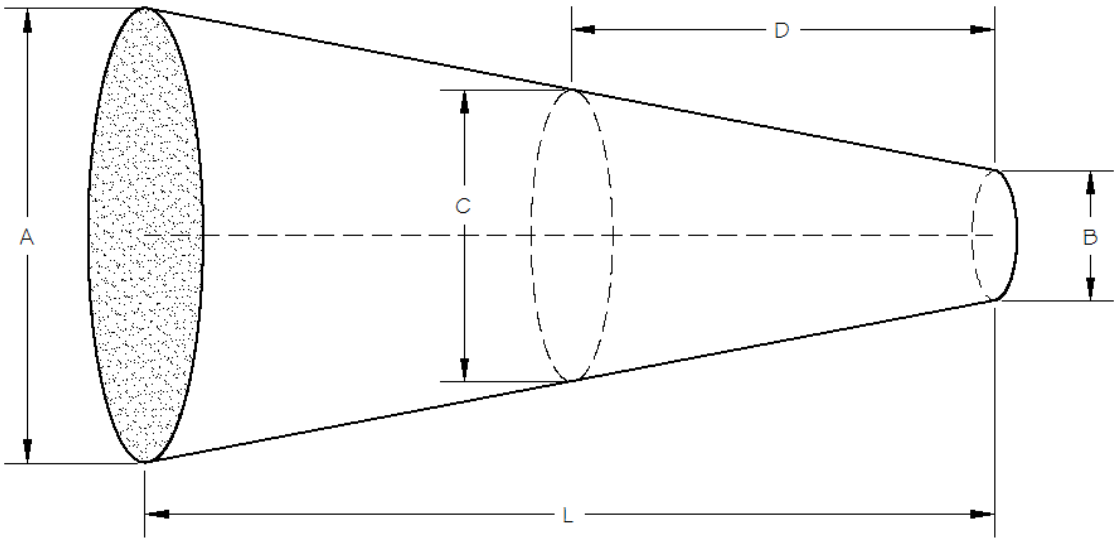

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$V_p$ = Voltage at the Primary side
$V_s$ = Voltage at the Secondary side
$N_p$ = Number of turns of wire at the Primary side
$N_s$ = Number of turns of wire at the Secondary side

Calculate the missing values in the table. Round all values to the nearest whole number when necessary.

| $V_p$ | $V_s$ | $N_p$ | $N_s$ |
|---|---|---|---|
| 240 V | 40 V | 32 | |
| 320 V | 24 V | | 14 |
| 65 V | | 25 | 4 |
| | 8 V | 20 | 12 |

22. Taper is the difference between the diameters at each end of a part of a given length. A reamer is a tapered drill bit that can bore a hole of diameter C if inserted to depth D. Use a proportion to fill in the missing values in the chart accurate to 3 decimal places.




| Length L | Diameter A | Diameter B | Depth D | Diameter C |
|---|---|---|---|---|
| 4” | 1.2” | .5” | 1.4” | see example |
| 5” | 1.5” | .75” | 2” | |
| 84 mm | 2.8 mm | 1.2 mm | | 2 mm |
| 9.5 cm | 4.2 cm | 3.2 cm | | 3.6 cm |

Example:

$$\frac{Taper_1}{Length_1} = \frac{Taper_2}{Length_2}$$

$$\frac{1.2-.5}{4} = \frac{T_2}{1.4}$$

$$\frac{.7}{4} = \frac{T_2}{1.4}$$

$1.4 \times .7 = 4 \times T_2$

$.98 = 4 \times T_2$

$.245 = T_2$

C - .5= .245 … C = .745”