1.4: Dimensional Analysis

Do not get intimidated by the title of this section, the concept is simple. The units (dimensions) for a number will help, rather than add to, your mathematical difficulties.

Students often have trouble remembering if area, with dimensions measured in feet, is labeled ft, ft^2 or ft^3. Pay attention to the units of the numbers and there is nothing to remember. Finding the area of a rectangular concrete porch that is 8ft x 9ft is 72, since the area of a rectangle is L x W. Notice that if you pay attention to the units you are also multiplying ft x ft which equals ft^2. The answer is 72 ft^2.

A typical concrete problem involves calculating the amount to order for a sidewalk shaped like a rectangular box. Suppose it is 4 inches deep, 6 feet wide and 100 feet long. If you do not pay attention to the units you cannot order the correct amount. To make matters worse, concrete is ordered by the cubic yard (yd^3). Analyzing the dimensions or units of a number is indispensable in mathematics.

Let's put these two concepts together to solve the sidewalk problem:

Example 1.4.1: Calculating concrete volume

Find the volume of concrete in yd^3 needed for a sidewalk that is 4 in deep by 6 ft wide by 100 ft long.

Solution:

You may remember that the volume of a box is L x W x H (see the appendix for common formulas). If you multiply 4 in x 6 ft x 100 ft, you would get 2400, which is wrong, and the units would be ft^2-in which do not make sense.

A cubic inch (in^3) is not only a unit; it is literally a cube that is 1in x 1in x 1in. Start by converting the dimensions of the sidewalk to inches. Since there are 12 inches in 1 foot, let the units and some basic algebra skills do the work. $6\text{ft} \bullet \frac{12\text{in}}{1\text{ft}}$ would change the units of 6 ft without changing the actual length. You multiply fractions straight across so $\frac{6\text{ft}}{1} \bullet \frac{12\text{in}}{1\text{ft}} = \frac{6\cancel{\text{ft}}}{1} \bullet \frac{12\text{in}}{1\cancel{\text{ft}}} = 72\text{in}$.

The idea is this, if you need the units of feet to change to inches then you would have to multiply by the unit $\left(\frac{\text{in}}{\text{ft}}\right)$. Similarly, 100 ft = 1200 in, so we have 4 in x 72 in x 1200 in = 345,600 in^3.

This is the correct answer but needs to be in yd^3. We would have to multiply by the unit $\frac{\text{yd}^3}{\text{in}^3}$ to change to yd^3. The appendix at the end of this text lists all the conversions you will need for this section. Observe that 46,656 in^3 = 1 yd^3. Therefore the problem becomes:

$\frac{345{,}600\text{in}^3}{1} \bullet \frac{1\text{yd}^3}{46{,}656\text{in}^3}$ notice that you are actually multiplying by 1 since 46,656 in^3 = 1 yd^3

$\frac{345{,}600\cancel{\text{in}^3}}{1} \bullet \frac{1\text{yd}^3}{46{,}656\cancel{\text{in}^3}}$ cubic inches "cancel" out leaving cubic yards and we are led to divide

$\frac{345{,}600\text{yd}^3}{46{,}656} \approx 7.4\text{ yd}^3$ simplify

Final Answer: Volume ≈ 7.4 yd^3 of concrete.

This technique can also be used to change to different types of units like ft^3 to gallons, as the following example illustrates:

Example 1.4.2: Changing from cubic feet to gallons

Volumes of liquid are commonly measured in gallons. Find the number of gallons needed to fill a 4 ft x 9 ft x 14 ft box.

Solution:

The volume of the box would be 504 ft^3 using the formula from the previous example. Changing the units to gallons (gal) from ft^3 would require us to multiply by the unit $\left(\frac{\text{gal}}{\text{ft}^3}\right)$. The appendix gives the conversion: 1 ft^3 = 7.5 gal.

$\frac{504\text{ft}^3}{1} \bullet \frac{7.5\text{ gal}}{1\text{ft}^3}$

$\frac{504\cancel{\text{ft}^3}}{1} \bullet \frac{7.5\text{ gal}}{1\cancel{\text{ft}^3}}$ cubic feet cancel out leaving gallons and we are led to multiply

504 x 7.5 gal = 3780 gallons simplify

Final Answer: A volume of 3780 gallons of water are needed to fill the box.

It may seem that multiplying 504 by 7.5 would change the answer, but remember that since 1 ft^3 = 7.5 gal that the fraction $\frac{7.5\text{ gal}}{1\text{ft}^3}$ actually equals 1. We are changing the units and the number without changing the actual amount of liquid.

This technique also helps convert back and forth from metric to standard units:

Example 1.4.3: Converting from metric to standard weight

A length of steel tubing is labeled to weigh 8700 grams, convert this weight to pounds to determine how much trouble it will be to lift.

Solution:

Changing the units to pounds (lbs) from grams (g) would require us to multiply by the unit $\left(\frac{\text{lbs}}{\text{g}}\right)$. The appendix gives the conversion: 1 lb = 453.6 g.

$\frac{8700\text{g}}{1} \bullet \frac{1\text{ lb}}{453.6\text{ g}}$

$\frac{8700\cancel{\text{g}}}{1} \bullet \frac{1\text{ lb}}{453.6\cancel{\text{ g}}}$ the grams cancel out leaving pounds and we are led to divide

$\frac{8700\text{ lbs}}{453.6}$ ≈ 19.2 pounds simplify

Final Answer: Weight ≈ 19.2 pounds, an easy size to work with, let’s carry three at a time!

Consider an example where there are multiple units to change:

Example 1.4.4: Converting a metric speed to miles per hour (MPH)

Convert a speed of 24 meters/sec to mph ($\frac{\text{miles}}{\text{hour}}$).

Solution:

Start with 24 m/s and multiply by conversion factors that change seconds into hours and meters into miles:

$\frac{24\text{ m}}{1\text{sec}} \bullet \frac{60\text{ sec}}{1\text{ min}} \bullet \frac{60\text{min}}{1\text{ hr}} \bullet \frac{3.281\text{ ft}}{1\text{ m}} \bullet \frac{1\text{ mile}}{5280\text{ ft}}$ conversions taken from the appendix

$\frac{24\ \cancel{\text{m}}}{1\ \cancel{\text{sec}}} \bullet \frac{60\ \cancel{\text{sec}}}{1\ \cancel{\text{min}}} \bullet \frac{60\ \cancel{\text{min}}}{1\text{ hr}} \bullet \frac{3.281\ \cancel{\text{ft}}}{1\ \cancel{\text{m}}} \bullet \frac{1\text{ mile}}{5280\ \cancel{\text{ft}}}$ the remaining units are $\frac{\text{miles}}{\text{hour}}$

$\frac{24*60*60*3.281\text{ mile}}{5280\text{ hr}} \approx 53.7$ mph we know exactly what to multiply and divide

Final Answer: Speed ≈ 53.7 mph

Note: Here is another possibility: $\frac{24\ \cancel{\text{m}}}{1\cancel{\text{sec}}} \bullet \frac{3600\ \cancel{\text{sec}}}{1\text{ hr}} \bullet \frac{100\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}} \bullet \frac{1\ \cancel{\text{in}}}{2.54\ \cancel{\text{cm}}} \bullet \frac{1\ \cancel{\text{ft}}}{12\ \cancel{\text{in}}} \bullet \frac{1\text{ mile}}{5280\ \cancel{\text{ft}}} \approx 53.7$ mph. The key is that the units lead you to the math that is required. This problem would be daunting if left entirely to common sense, unless you are uncommonly sensible.

Section 1.4: Dimensional Analysis

Refer to the appendix for common conversions

Length Conversions

1. Convert 38 inches into a measurement in centimeters, rounded to one decimal place.

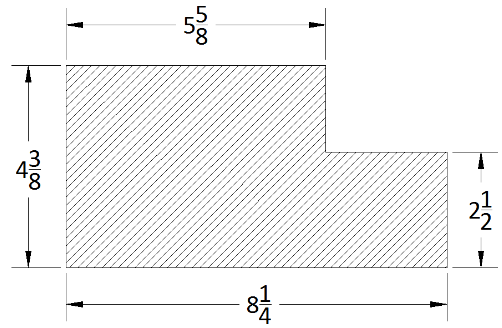
2. Convert 157 centimeters into a measurement in inches, rounded to the nearest 16th of an inch.

3. Convert 51 inches into a measurement in millimeters, rounded to one decimal place.

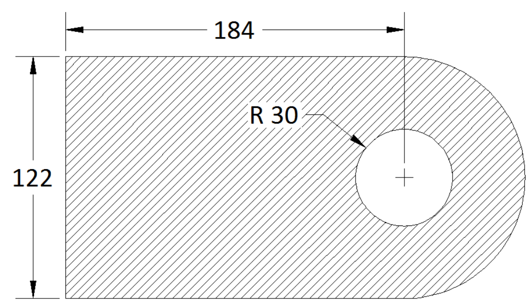
4. Convert 9 meters into a measurement in feet and inches, rounded to the nearest 16th of an inch.

5. Convert 9456 feet into a measurement in miles, rounded to two decimal places.

6. The measurements in the drawing are given in inches. Convert each dimension to centimeters, rounded to one decimal place.



7. The measurements in the drawing are given in millimeters. Convert each dimension to inches rounded to the nearest 16th of an inch.



Area Conversions

8. Convert 17 square inches into a measurement in square centimeters, rounded to two decimal places.

9. Convert 236 square inches into a measurement in square feet, rounded to two decimal places.

10. A lot in a subdivision is listed as .32 acres, convert .32 acres to a measurement in square feet, rounded to the nearest square foot.

11. Convert 78 square feet into a measurement in square yards, rounded to two decimal places.

12. Convert 789 square inches into a measurement in square meters, rounded to two decimal places.

13. Convert five square meters into a measurement in square feet, rounded to two decimal places.

Volume Conversions

14. Convert four cubic feet into a measurement in cubic inches.

15. Convert five cubic feet into a measurement in gallons, rounded to two decimal places.

16. Convert 167 cubic feet into a measurement in cubic yards, rounded to two decimal places.

17. Convert 183,487 cubic inches into a measurement in cubic yards, rounded to one decimal place.

Rate Conversions

18. Convert a rate of 18 feet per second into miles per hour (MPH), rounded to one decimal place.

19. Convert 23 gallons per minute (GPM) into cubic feet per day, rounded to the nearest whole number.

20. A tree grows 3/8" per day. Convert this growth rate into feet per year, rounded to two decimal places.

21. Convert 27 miles per hour (MPH) into meters per second, rounded to one decimal place.

22. A paint striper covers 100 meters in 28 seconds. Convert this speed to MPH, rounded to one decimal place.

23. An assembly line manufactures I-beams at a rate of 58 feet per second. Convert this speed to miles per hour (MPH), rounded to one decimal place.

Weight Conversions

24. Use the chart of weights of common metals to make the conversions rounded to one decimal place:

Weights of Common Metals

Material	Weight (oz/in^3)
Magnesium	1.006
Aluminum	1.561
Titanium	2.641
Zinc	4.104
Stainless Steel	4.538
Copper	5.180
Tungsten	11.157
Gold	11.169

a. Convert the weight of Copper to ounces per cubic foot.

b. Convert the weight of Tungsten to grams per cubic inch.

c. Convert the weight of Magnesium to grams per cubic foot.

d. Convert the weight of Stainless steel to pounds per cubic foot.

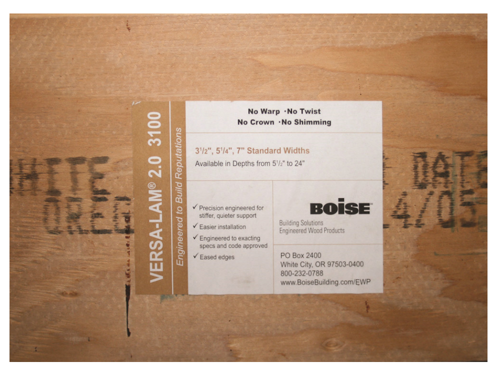
e. Convert the weight of Titanium to grams per cubic centimeter.

25. Find the weight of a five gallon bucket of concrete measured in pounds, rounded to one decimal place. Concrete weighs 3,915 pounds per cubic yard.

26. Find the weight of a four cubic foot wheel barrow filled with concrete measured in pounds, rounded to the nearest pound. Concrete weighs 3,915 pounds per cubic yard.

27. Find the weight of a 3,072 cubic inch Douglas fir header measured in pounds, rounded to one decimal place. Douglas fir weighs 38 pounds per cubic foot.

28. Find the weight of a 16 cubic foot laminated veneer lumber (LVL) beam measured in pounds, rounded to the nearest pound. LVL beams weigh .34 ounces per cubic inch.



29. Find the weight of two 5 gallon buckets of water measured in pounds, rounded to one decimal place. Water weighs 62.4 pounds per cubic foot.