1.5: Order of Operations

We finish the chapter with the order of operations used without explanation in the introduction to this book. It is impossible to understand algebra in a meaningful way without a deep understanding of this foundational concept. The order is easily memorized with the acronym PEMDAS, which is often expanded to Please Excuse My Dear Aunt Sally, as a mnemonic device.

1. **P**arentheses
2. **E**xponents
3. **M**ultiply/**D**ivide
4. **A**dd/**S**ubtract

For me this is a bit uninspired for such an important concept so my personal favorite is: **P**eople **E**njoying **M**ath **D**isplay **A**wkward **S**ymptoms. It is acceptable to have a little fun while learning math, in fact, if you are not having fun you are doing something wrong. To deepen your grasp of this pivotal concept, let's take a minute and discuss the reason for this order.

I am embarrassed to admit that I graduated from college with a math degree without knowing what this reason is. Addition is the foundational math operation; it can be argued that addition is the only math operation. Consider the problem 3 x 4, you have memorized that it is 12 to save time. The answer is 12 because 3 x 4 means three-fours or 4 + 4 + 4. We invent multiplication to speed up the process when repeatedly adding the same number. For example, you buy 42 studs to build a wall costing $3.52 each. The total cost is found by adding $3.52 to itself 42 times. Multiplication thankfully speeds this process up, but the fact remains that multiplication is merely a useful technique for adding quickly.

If a formula requires you to find 3 x 6 + 5, understanding that it is shorthand for 6 + 6 + 6 + 5 which equals 23, implies that you must multiply first or you won't get it right. If you add before multiplying, cashier would charge you $33 instead of $23 for three $6 pipes and one $5 saw. The order does not result from my dear Aunt Sally. She became famous later merely for being a dear woman whose name started with an S. What does she have to say about exponents preceding multiplication?

Again the issue is resolved in the meaning of an exponent. 5^3 means 5 x 5 x 5. Exponents are shorthand for multiplication. Consider the expression 5 x 4 + 3^2.

5 x 4 + 3 x 3	using the definition of exponents
4 + 4 + 4 + 4 + 4 + 3 + 3 + 3	using the definition of multiplication
29	simplify (notice the complex expression is all addition when expanded to show its meaning)

Multiplication and exponents make it convenient for writing math expressions that would otherwise be very cumbersome, but the shorthand comes at the price of needing some skill to interpret it.

I have made no mention of subtraction and division. This is because they are simply invented as inverse operations. Division is just the inverse of multiplication and subtraction is the inverse of addition. It is important to understand that multiplication does not precede division; in fact, these operations must be performed from left to right as you would read a sentence. Similarly, addition and subtraction operations must also be performed working left to right.

Indeed then, in a sense, there is only one operation in math, addition. All other operations are either inverse operations or shortcuts. Although I would not recommend shouting this revelation the next time you are in a crowd for fear of being misunderstood, the fact should solidify your grasp on the order of operations. It should also drive home the fact that mathematics is not the memorization of useless and unrelated facts. It can be understood at a foundational level and applied to life in useful and time-saving ways.

Finally, parentheses are first in the order simply as a grouping symbol with the express meaning "do this first". It must be noted that large division bars in an expression like $\frac{3+4\text{x}6}{5^2-7}$ imply parentheses, so that you must simplify the top (numerator) and the bottom (denominator) before performing the division. Most calculators allow for entering an entire expression like this, but do not have a large division symbol and so must be entered as $(3+4*6)/(5\hat{}2-7)$. It must also be noted that large square root symbols carry the same implication. An expression like $\sqrt{1+4^2}$ requires that the inside of the square root be determined as 17 before taking the square root. Again you must enter $\surd(1+4\hat{}2\ )$ into a calculator to get the correct answer since there is no large square root symbol.

Consider a simple example involving money growing at an interest rate:

Example 1.5.1: Calculating simple interest

Solve for the final amount of money (A), if the principal (P) = $460, the interest rate (r) = 8.3%, and the time (t) = 10 years. Use the formula: $\mathrm{A} = \mathrm{P} + \mathrm{Prt}$.

Solution:

A = $460 + 460 * .083 * 10$	substitute the numbers into the formula
A = 460 + 381.8	multiplication is before addition and is done from left to right
A = 841.8	simplify

Final Answer: The amount is $841.80.

Consider an example involving the cornering force that a vehicle exerts on its passengers as a function of the radius of the turn and the speed of the vehicle:

Example 1.5.2: Calculating the force experienced when cornering in a vehicle

Solve for the force (F) measured in g's, if you know the velocity (v) = 48 mph and the radius (r) = 24 feet.

Use the formula: $F = \frac{v^2}{14.957r}$. Round to the nearest tenth.

Solution:

$F = \frac{48^2}{14.957*24}$ substitute the numbers into the formula

$F = \frac{2304}{14.957*24}$ exponents are first, (take time here to learn your calculator, enter: 48 ^ 2 or 48 x^y 2, depending on the type of calculator that you have, this will save time when the exponent is larger than 2)

$F = \frac{2304}{358.968}$ remember the large division implies a parentheses so the top and bottom must be simplified first (if you enter 2304 / 14.957 x 24 in your calculator it will be wrong)

F ≈ 6.4 simplify

Final Answer: The cornering force is F ≈ 6.4 g's, meaning about six and half times the force of gravity.

Consider a more realistic and complex example of money growing at a monthly interest rate:

Example 1.5.3: Compound interest formula

Solve for the amount (A) if the principal (P) = \$580, the interest rate (r) = 7.2%, and the time (M) = 42 months. Use the formula: $A = P(1 + \frac{r}{12})^M$.

Solution:

A = $580\left(1 + \frac{.072}{12}\right)^{42}$ substitute the numbers into the formula

A = $580 * 1.006^{42}$ parentheses are first and $1 + \frac{.072}{12}$ simplifies to 1.006 when dividing before adding

A ≈ $580 * 1.286$ exponents are next, 1.006^{42} simplifies to 1.286 (rounded to the thousandth place and I hope you figured out how to enter this in your calculator)

A ≈ 745.88 simplify

Final Answer: A deposit of \$580 at 7.2% APR for 42 months would grow to \$745.88.

Note: A bank would report the amount as \$745.66 since they would not round until the end.

Consider a formula used to calculate roofing for a house, involving a large square root sign:

Example 1.5.4: Calculating square feet of roofing

Solve for the number of square feet of roofing (R), if the slope of the roof (s) is $\frac{5}{12}$ and the floor area (A) = 1860 ft^2. Use the formula: $R = A\sqrt{1+s^2}$. Round to the nearest square foot.

Solution:

R = $1860\sqrt{1+\left(\frac{5}{12}\right)^2}$	substitute the numbers into the formula
R ≈ $1860\sqrt{1+(.417)^2}$	parentheses first and rounding to the thousandth place
R ≈ $1860\sqrt{1.174}$	The large square root implies a parenthesis and so $1+.417^2$ must be simplified. Squaring .417 gives .174. Adding 1 results in 1.174.
R ≈ $1860 * 1.08$	a square root is actually an exponent of .5, so the square root is before multiplication in the order of operations
R ≈ 2009	simplify

Final Answer: R ≈ 2009 square feet of roofing. Note: You will get R = 2015 if you do not round until the end. Rounding multiple times in a calculation can result in an answer that is remarkably inaccurate. This formula is a remarkable simplification of the difficult process it would be to calculate all the areas of each rectangle, triangle, parallelogram, and trapezoid that often compose a roof. A person who knows algebra can get more accurate material calculations and save a lot of time in the process.

For the last example we consider a remarkably complicated engineering formula for the deflection (bending) of a cantilevered beam. I will spare you the meaning of each letter and the complexity involved with more realistic numbers.

Example 1.5.5: Calculating beam deflection

Solve for D if w = 4, n = 2, L = 3, E = 5, and I = 6. Use the formula:
$D = \frac{w(n^4-4nL^3+3L^4)}{24EI}$.

Solution:

D = $\frac{4(2^4-4*2*3^3+3*3^4)}{24*5*6}$	substitute the numbers into the formula
D = $\frac{4(16-4*2*27+3*81)}{24*5*6}$	exponents in the parentheses first
D = $\frac{4(16-216+243)}{24*5*6}$	multiplications inside the parenthesis next
D = $\frac{4*43}{24*5*6}$	finish the parenthesis working left to right
D = $\frac{172}{720}$	since the large division sign implies a separate parentheses top and bottom, simplify the top and bottom separately
D ≈ .24	simplify

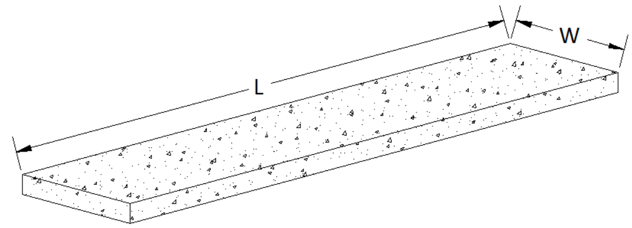
Final Answer: D ≈ .24; the beam will bend or deflect approximately $\frac{1}{4}$ inch.

Section 1.5: Order of Operations

1. The volume (V) of concrete for a driveway or sidewalk is often estimated by $V = \frac{LW}{80}$.

   Where V = volume measured in cubic yards, L = length measured in feet, and W = width measured in feet.

   Find the volume of concrete needed for a driveway that is 24' wide and 12' long.



2. The volume (V) of concrete for a driveway or sidewalk is often estimated by $V = \frac{LW}{80}$.

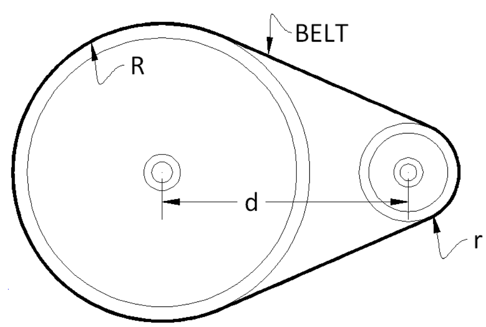
   Where V = volume measured in cubic yards, L = length measured in feet, and W = width measured in feet.

   Find the volume of concrete needed for a sidewalk that is 5 feet wide and 124 feet long.

3. An approximation of the belt length (L) in a motor is $L = \pi(R + r) + 2d$.

   R = radius of the larger pulley, r = radius of the smaller pulley, and d = distance between the pulleys. Pi (π) ≈ 3.14 but most calculators have a pi button that should be used instead since it is quicker and more accurate.

   Find the length of the belt rounded to one decimal place, if R = 16 cm, r = 7 cm, d = 28 cm.



4. The carburetor size formula is $C = \frac{dRV}{3456}$.

   C = cubic flow modification (CFM), d = piston displacement, R = engine revolutions per minute (RPM), and V = volumetric efficiency.

   Find the CFM for a 3300 RPM engine with a 5.2-inch piston displacement and a volumetric efficiency of 124%, rounded to one decimal place.

5. The torque formula is $T = \frac{5252H}{R}$.

   T = torque, H = horsepower, and R = engine revolutions per minute (RPM).

   Find the torque for a 2300 RPM engine with 840 horsepower, rounded to one decimal place.

6. In electronics, Power (P) is $P = RI^2$.

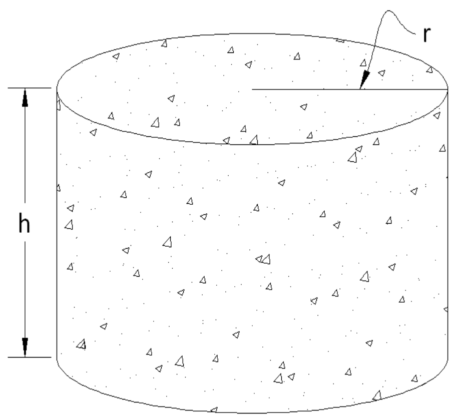
   P = power measured in watts, R = resistance measured in ohms, and I = current measured in amps.

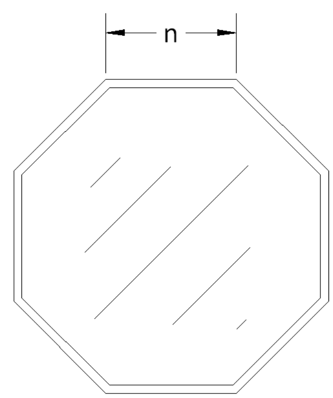
   Find the power consumed by an 8-ohm resistor with a 9-amp current passing through it.

7. The volume of cylindrical footing (V) is $V = \pi r^2 h$.

   H = height and r = radius.

   Find the volume rounded to one decimal place, if h = 14" and r = 8".

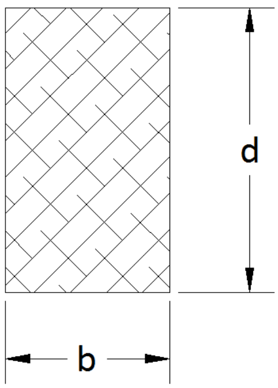




8. The area of a regular octagonal window (A) is $A = 4.828n^2$.

   Find the area of an octagonal window with a side of 12" rounded to one decimal place.

9. The moment of inertia (I) of a beam is $I = \frac{bd^3}{12}$.



Note: Moment of inertia is a measure of a beam's ffectiveness at resisting bending based on its cross-ectional shape.

= moment of inertia of the beam measured in inches^4, = width of the beam measured in inches and d = height f the beam measured in inches.

ind the moment of inertia of a beam rounded to one ecimal place, if b = 3.5" and d = 14".

10. The speed of a car is $S = \frac{DR}{336G}$.

S = speed in miles per hour (MPH), D = tire diameter in inches, R = engine revolutions per minute (RPM), and G = gear ratio.

Find the speed of a car with 26 inch diameter tires, a 3400 RPM engine, and a gear ratio of 3.5, rounded to the nearest MPH.

11. The reactance offered by a capacitor in electronics is $X = \frac{1}{2\pi fC}$.

X = reactance measured in ohms, f = frequency measured in cycles per second (hertz), C = capacitor size measured in farads.

Find the reactance for a capacitor in a circuit with a frequency of 60 hertz and a capacitor size of .00012 farads, rounded to three decimal places.

12. The voltage drop in an electrical wire is $V = \frac{2LIR}{1000}$.

AWG	R
16	4.884
14	3.072
12	1.932
10	1.215
8	.764
6	.481
4	.302

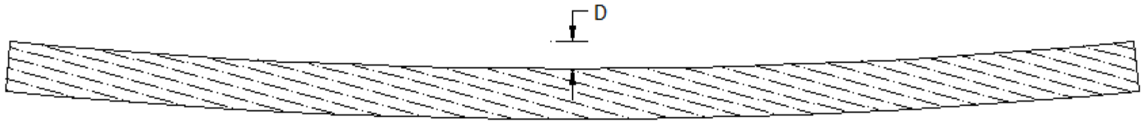
V = voltage drop measured in volts, L = length of the wire measured in feet, I = current measured in amps and R = resistance in the wire measured in ohms.

Use the table at the right to determine the voltage drop in a 350 foot #12 AWG electrical cord attached to a saw drawing 13 amps of current. Round to one decimal place.

Recalculate if the cord size is increased to #10 AWG.

13. The point load deflection (D) of a beam is $D = \frac{PL^3}{48EI}$.

Note: Deflection is simply a measurement of the amount of bend in a beam.



D = deflection measured in inches, P = weight on the beam measured in pounds, L = length of the beam measured in inches, E = elasticity of the beam measured in pounds per square inch (PSI), and I = moment of inertia of the beam measured in inches^4.

Find the deflection of a beam rounded to one decimal place if L = 216, P = 4500, E = 1,800,000, and I = 432.

14. The uniform load deflection (D) of a beam is $D = \frac{5PL^4}{384EI}$.

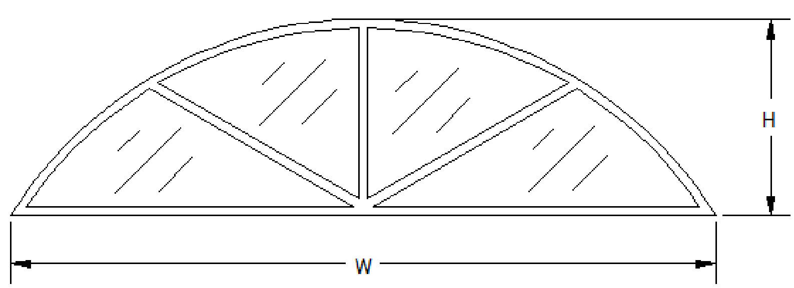
Note: Deflection is simply a measurement of the amount of bend in a beam.

D = deflection measured in inches, P = weight on the beam measured in pounds, L = length of the beam measured in inches, E = elasticity of the beam measured in pounds per square inch (PSI), and I = moment of inertia of the beam measured in inches^4.

Find the deflection of a beam rounded to one decimal place if L = 168 inches, P = 358 pounds, E = 2,000,000 psi, and I = 968 inches^4.

15. The formula to calculate the radius (R) of an arch window is $R = \frac{W^2 + 4H^2}{8H}$.

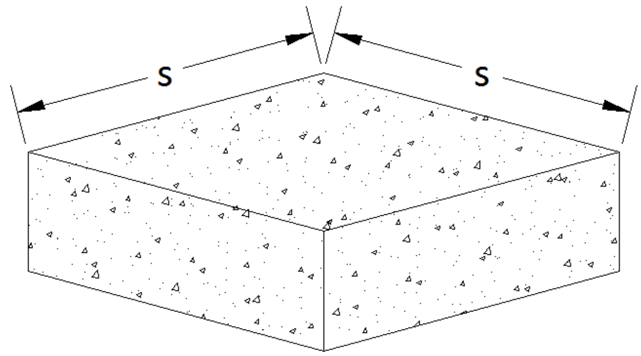
W = width of the window & H = height of the window.



Find the radius of an arch window accurate to the 16^{th} of an inch, which has a width of 42 inches and a height of 12 inches.

16. The formula to calculate size (S in inches) of a square footing is $S = 12\sqrt{\frac{W}{B}}$.

W = weight on the footing (in pounds) & B = soil bearing capacity in pounds per square foot (PSF).

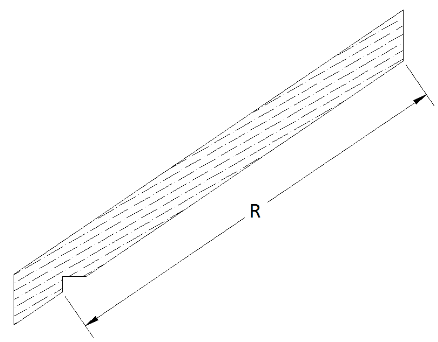


Find the size (S) of footing necessary to hold a weight of 7400 pounds that sits on soil able to bear 1500 psf. Round your answer to one decimal place.

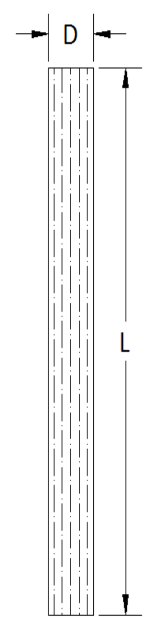
17. The length of a rafter (R) can be calculated using the formula: $R = \frac{W}{2}\sqrt{S^2 + 1}$.

R = length of the rafter measured in inches, W = width of the building measured in inches, and S = slope of the roof.

Find the length of a rafter for a building that is 312" wide and has a slope of 7/12, rounded to the nearest 16^{th} of an inch.



18. The allowable stress (S) on a post is $S = \frac{3ED^2}{10L^2}$.



S = allowable stress measured in pounds per square inch (PSI), D = dimension of the post measured in inches, L = length of the post measured in inches, and E = elasticity of the beam measured in pounds per square inch (PSI).

Find the allowable stress on a post rounded to one decimal place, if L = 120 inches, E = 1,100,000 psi, D = 3.5 inches.

19. The Cornering Force (F) that a vehicle exerts on its passengers is $F = \frac{v^2}{32r}$.

F is measured in g's (one g is the force of gravity of the earth), v = velocity measured in feet per second and r = radius measured in feet.

Find the cornering force of a car traveling 82 feet per second around a corner of radius 48 feet, rounded to one decimal place.

20. The Engine Displacement (D) is $D = \frac{\pi b^2 sc}{4}$.

D = engine displacement measured in cubic centimeters, b = bore (diameter of the cylinder) measured in centimeters, s = stroke (distance that the piston travels) measured in centimeters, and c = number of cylinders.

Find the displacement of an 8 cylinder engine with a 2.8-cm bore and a 4.4-cm stroke, rounded to one decimal place.

21. The Exhaust Header Tubing Length (L) is $L = \frac{1900D}{d^2 R}$.

L = length measured in inches, D = displacement measured in cubic inches, d = exhaust head diameter measured in inches, and R = revolutions per minute (RPM).

Find L, if D = 350 in^3, d = 3 in, and R = 2200 RPM's, rounded to the nearest inch.

22. In electronics, Power (P) is $P = \frac{E^2}{R}$.

P = power measured in watts, R = resistance measured in ohms, and E = voltage measured in volts.

Find the power consumed by 120 volt electricity passing through a circuit with .6 ohms of resistance.

23. Roofing for a house can be ordered by the square (1 square = $100ft^2$). The formula for calculating the number of squares of roofing for a house is $R = \frac{A\sqrt{1+S^2}}{100}$.

    R = number of squares, A = area or square footage of the floor of the house, and S = slope of the roof. Find the number of squares of roofing for a $1950ft^2$ house with a roof slope of 9/12, rounded up to the nearest whole number.

24. The formula for horsepower is $H = W\left(\frac{S}{234}\right)^3$.

    H = horsepower, W = weight in pounds, and S = speed in MPH.

    Calculate the horsepower for a car that weighs 2740 pounds and is capable of 106 miles per hour (MPH), rounded to the nearest horse. Horses, as you know, are weak with decimals.

25. The formula for resistance in a parallel electrical circuit is $R_t = \frac{R_1 R_2}{R_1 + R_2}$.

    R_t = total resistance, R_1= resistance one, and R_2= resistance two.

    Find the total resistance if R_1 = 460 Ω and R_2 = 720 Ω, rounded to one decimal place. Note: Ω is an electrical symbol for ohms.

26. The formula for speed is $S = 234\left(\frac{H}{W}\right)^{.333}$.

    H = horsepower, W = weight in pounds, and S = speed in MPH.

    Calculate the speed for a car that weighs 2160 pounds with 712 horse power, rounded to the nearest MPH.

27. The formula to calculate impedance in an RL circuit is: $Z = \sqrt{R^2 + X^2}$.
    Z = impedance measured in ohms
    R = resistance measured in ohms
    X = reactance measured in ohms

    Calculate the impedance in a circuit with 2.3 kΩ of resistance and a reactance of 5.4 kΩ, rounded to the nearest tenth of a kΩ.

28. The formula to calculate impedance in an RL circuit is: $Z = \sqrt{R^2 + X^2}$.
    Z = impedance measured in ohms
    R = resistance measured in ohms
    X = reactance measured in ohms

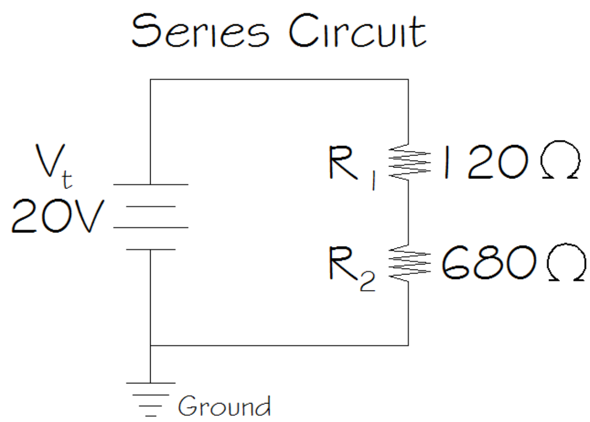
    Calculate the resistance in a circuit with 6.2 kΩ of impedance and a reactance of 4.1 kΩ, rounded to the nearest tenth of a kΩ.

29. The formula to calculate inductive reactance is: $X = 2\pi fL$
    X = inductive reactance measured in ohms
    f = frequency measured in hertz (Hz)
    L = inductance measured in henrys (H)

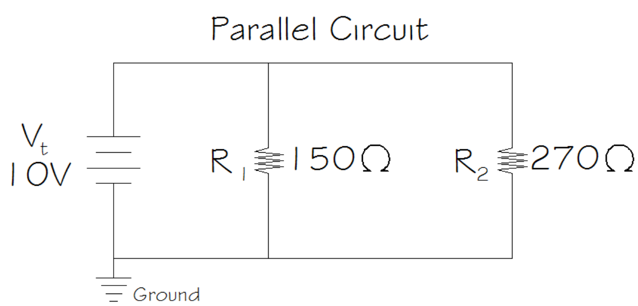
    Calculate the inductive reactance for an AC circuit with a frequency of 3 kHz and an inductance of 12.3 mH. Note: kHz and mH have metric prefixes that must be considered. Round to the nearest whole number.

30. Use the Voltage Divider Formulas to calculate voltages V_1 and V_2 at each resistor R_1 and R_2.
    V_1 = voltage at R_1
    V_2 = voltage at R_2
    R_1 = resistance at R_1 measured in ohms
    R_2 = resistance at R_2 measured in ohms
    R_t = total resistance measured in ohms

    $V_1 = V_t\left(\frac{R_1}{R_t}\right)$ & $V_2 = V_t\left(\frac{R_2}{R_t}\right)$ also note that $R_t = R_1 + R_2$

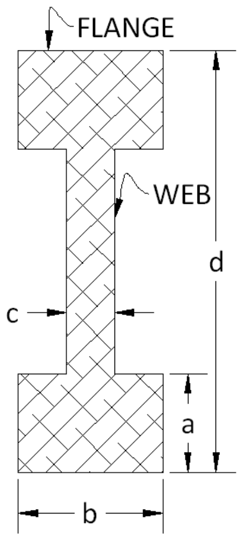


31. Use the Current Divider Formulas to calculate currents I_1 and I_2 at each resistor R_1 and R_2. Answer in amps rounded to 2 decimal places.
    I_1 = amperage at R_1
    I_2 = amperage at R_2
    R_1 = resistance at R_1 measured in ohms
    R_2 = resistance at R_2 measured in ohms
    R_t = total resistance measured in ohms



$I_1 = I_t\left(\frac{R_t}{R_1}\right)$ & $I_2 = I_t\left(\frac{R_t}{R_2}\right)$ also note that $R_t = \frac{R_1 R_2}{R_1 + R_2}$ and $I_t = \frac{V_t}{R_t}$

33. **Challenge Problem:** the moment of inertia (I) of an I-joist is $I = \frac{bd^3 - (b-c)(d-2a)^3}{12}$.



Note: Moment of inertia is a measure of a beam's effectiveness at resisting bending based on its cross-sectional shape.

I = moment of inertia of the beam measured in $inches^4$, a = flange thickness measured in inches, b = width of the flange measured in inches, c = web thickness measured in inches, and d = height of the joist measured in inches.

Find the moment of inertia of Boise Cascade's 5000 series BCI where a = 2-1/2", b = 1-1/8", c = 3/8" and d = 9-1/2". Round your answer to one decimal place.

34. **Challenge Problem:** A keyway is often cut in a cylindrical shaft in machining to lock two parts together. Achieving the desired depth (k) can be accomplished by calculating the depth of cut (d). The formula for calculating the depth (d) of the cut is

$$d = \frac{2k + D - \sqrt{D^2 - w^2}}{2}.$$

Find d if k = .250, D = 2.875, and w = .625 (all measures are in decimal inches). Round your answer to the nearest thousandth of an inch.